MATHEMATICS

For Junior High School

BASIC 9

STRAND 1: NUMBER

SUB-STRAND 1: NUMBER AND NUMERATION SYSTEMS

LESSON 1

In this lesson, we shall express integers to some significant figures and express decimal numbers to a given number of decimal places.

Significant figures

Significant figures are the digits in a number that contributes to the accuracy of the number. The first significant figure is the first non-zero digit of a number, the second non-zero digit is the second significant figure, and so on. Significant figures are used to establish the number figures are used to establish the number which is presented in the form of digits. A significant figure could be to the right or left of the decimal point.

Rules for deciding the number of significant figures

1. All non-zero digits are significant e.g., 3.4567 has 5 significant figures, 0.123 has 3 significant figures.
2. Zeros between non-zero digits are significant:
   e.g. 200006 has 6 significant figures.
   4.0012 has 5 significant figures.
3. Zeros to the left of non-zero digits are not significant:
   e.g. 0.00002 has 1 significant digit.
   0.0321 has 3 significant figures.
4. Zeros to the right of decimal figures in a number are significant.
   e.g. 0.0415 has 3 significant figures.
   0.3100000 has 7 significant figures.

Rules for rounding off numbers

1. If the digit to be dropped is greater than or equal to 5, the last digit to be maintained is increased by one.
   e.g. 70.9802 = 71.0 (to 3 significant figures).
2. If the digit to be dropped is less than 5, the last digit to be maintained is left as it is.
   e.g. 0.247390= 0.247 (to 3 significant figures).
   0.7005447 = 0.7005 (to 4 significant figures).

Worked examples

Example 1:

Round 3729 to one significant figure.

Solution

Step 1. Locate the significant figure for the degree of accuracy required. The first non-zero digit is the first significant figure. For 3729, the first non-zero digit is 3.

Therefore, 3 is the first significant figure.

Step 2: Look at the next digit to the right. Is it 5 or more?
7 is more than 5.

Step 3: If it is 5 or more, round up by adding 1 to the previous digit. If it is less than 5, round down by keeping the previous digit the same. If the degree of accuracy is 10 or more, fill in zeros to make the number the correct size.

As 7 is more than 5, we round up. Add 1 to the 3, which gives us 4. The first significant figure represents thousands, so we must add three zeros to make it the correct size.

Therefore, 3729= 4000 (to 1 significant figure).

Example 2:

Round 36.582 to three significant figures.

Solutions

Step 1: Locate the significant figure for the degree of accuracy required. The first non-zero digit is the first significant figure. Therefore, the first 3 significant digits are 36.5

Step 2: Look at the next digit to the right of the last digit 5. If it is 5 or more, increase the third significant digit by 1 otherwise, leave it as it is.

In 36.582, 8 is more than 5, so increasing 5 by 1, gives 6.

Therefore, 36.582 =36.6 (3 significant figures).

Examples 3:

Round 0.83745 to 3 significant figures.

Solution

The first three significant figures give 0.837. But the next digit is 4 which is less than 5. So, we do not add anything to 7. Therefore, 0.83745= 0.837 (to 3 significant figures).

Example 4:

Round 3126879 to six significant figures

Solutions

The first six significant digits give 312687, but the next digit after 7 is 9 which is more than 5. So, add 1 to 7 to give 8 and replace the last digit 9 with 0 to maintain the place value of the digits.

Therefore, 3126879 = 3126880 to (6 significant figures).

Examples 5:

Round 348200 to three significant figures.

Solution

The first three significant figures give 348 but the next digit after 8 is 2 which is less than 5, so we add nothing to 8 and replace the last three digits with zeros to maintain the place value of the digits.

Therefore, 348200= 348000 to (3 significant figures).

Exercise 1

1. Round 63740 to one significant figure.
2. Round the following numbers to the indicated significant figures:
   a) 0.8379 to three significant figures.
   b) 62460 to three significant figures.
   c) 87.65 to two significant figures.
   d) 3:470114924 to three significant figures.
   e) 0.0045189 to three significant figures.
   f) 746893 to two significant figures.
   g) 0.08387 to one significant figure.
   h) 40.7283 to three significant figures.
   i) 0.0005897 to three significant figures.

Exercise 2

Round each of the following numbers to significant figures indicated.

1) 39.9748km to three significant figures.
2) 0.00025 to one significant figure.
3) 0.003858 to three significant figures.
4) 0.000344 to two significant figures.
5) 48371 to four significant figures.
6) 0.32076m to three significant figures.
7) 432.425 to five significant figures.
8) 2.70612 to three significant figures.
9) 0.000378 to one significant figure.
10) 40004650 to five significant figures.

Exercise 3

Express 587836124 to

a) Five significant figures.
b) Four significant figures.
c) Three significant figures.
d) Two significant figures.
e) One significant figure.

Expressing decimal numbers to a given number of decimal places

The decimal numeral system is the standard system for denoting integer and non-integer numbers. It is an extension of the non-integer numbers of the Hindu-Arabic system. A decimal number refers to a number that uses a decimal point followed by digits that show a value smaller than one. For example, 23.5 (twenty-three point five). Decimal numbers are used in everyday life such as currency and many more. Correcting a decimal number to some decimal places enables us to approximate the number.

Consider the following steps in rounding decimal numbers to a given number of decimal places.

1. Look at the first digit after the decimal point if rounding to one decimal place or the second digit for two decimal places.
2. Draw a vertical line to the right of the place digit that is required.
3. Look at the next digit.
4. If it is 5 or more, increase the previous digit by one.
5. Remove all the digits after the decimal point. The left-out number is the desired answer.

Example 1:

Round 854.57 to the nearest whole number.

Solution

Step 1: The whole number is 854

Step 2: The number in the tenth place is 5. SO, increase the digit in the one's place by one.

Therefore, 854.57 = 855 (to the nearest whole number).

Example 2:

Round 624.44 to the nearest tenths.

Solution

Step 1: The number in the tenth place is 4.

Step 2: The digit in the hundredth place is 4 which is less than 5. So, add nothing to the digit in the tenth place.

Therefore, 624.44 = 624.4 or 624.40

Example 3:

Emmanuel weights 28.63kg. What is the weight to the nearest kg?

Solution

The nearest kg is the same as the nearest whole number. The whole number is 28.

The digit in the tenth place is 6, so increase the digit in the one's place by one.

Therefore 28.63kg = 29 kg to the nearest kg.

Example 4:

The depth of Lake Volta is 1280.267m. what is the depth of the lake to the nearest hundredths?

Solution

The digit in the hundredth place is 6. The digit after the 6 is 7 which is more than 5. So, increase the digit in the hundredth place by one.

Therefore 1280.267m = 1280.27m to the nearest hundredths.

Example 5:

Round 5.1461 to the nearest hundredths

Solution

The digit in the hundredth place is 4. So, increase 4 by 1. Therefore, 5.1461 = 5.15 to (the nearest hundredths).

Take note

1. The nearest tenth is the same as one decimal place.
2. The nearest hundredth is the same as two decimal places.
3. The nearest thousandth is the same as three decimal places.
4. The nearest ten-thousandth is the same as four decimal places etc.

Exercise 4

1. Round the following numbers
   a. 2.176 to the nearest tenth.
   b. 316.94 to the nearest whole number.
   c. 16.1329 to the nearest thousandth.
   d. 413.76 to the nearest tenth.
   e. 0.01615 to the nearest hundredth.
   f. 3.4167 to the nearest tenth.
2. Correct 4687.02 to one decimal place.
3. Calculate, correct to two decimal places 0.610.8
4. Correct 0.003858 to four decimal places.
5. Correct 48947.2547 to the nearest hundred.
6. Correct 0.024561 to five decimal places.
7. Write GHd35632.00, correct to the nearest thousand cedis.
8. Write 78910, correct to the nearest thousand.

Exercise 5

1. Write 98475.6947, correct to
   a) Three decimal places.
   b) Two decimal places.
   c) One decimal place.
   d) The nearest ten.
   e) The nearest thousand.
2. Evaluate 0.25 x 0.06, and correct to three decimal places.
3. Correct 0.02783 to three decimal places.
4. Correct 48947.2547 to the nearest hundred.

LESSON 2

Real-life problems relating to place value

In this lesson, we shall create real-life problems and write solutions to them. Let us consider the following problems:

Example 1:

Find the difference between the place value and face value of 5 in the numeral 9263504.

Solution

The place value of the digit 5 in 9263504 is 500

The face value of digit 5 in 9263504 is 5.

Therefore, the difference is 500 -5 = 495.

Example 2:

Form a number with 7 at the thousands place, 9 at the once place, 2 at the tens place, 4 at ten the thousands place, 3 at the hundreds place, and 6 at the hundred thousands place.

Solution

Draw the place value chart

HT	TT	Th	H	Tens	Ones
6	4	7	3	2	9

Therefore, the number is 647,329

Example 3:

I am a numeral with 2 at the ten million place, 4 at the ten thousand place, 1 at the one million place, 3 at the hundred thousand place, 9 at the one's place, 8 at the hundreds place, 5 at the tens place, and 7 at the thousands place. What number am I?

Solution

Draw a place value chart

TM	M	HT	TT	TH	H	T	O
2	1	3	4	7	8	5	9

Therefore, the number is 21,347,859

Example 4:

write the smallest 5- digit number.

a) Having 5 different digits.
b) Having 7 at hundreds place.
c) Having 9 at thousands place.

Solution

a) The smallest 5 different digits are 0, 1, 2, 3, 4

Therefore, the smallest 5-digit number having 5 different digits is 10234

b) The smallest 5-digit number having 7 at the hundreds place is 10723.

c) The smallest 5-digit number having 9 at thousands place is19023.

Example 5:

Write the largest 5- digit number.

a) Having 5 different digits
b) Having 2 in the hundreds place.
c) Having 1 in the thousands place.

Solution

a) The largest 5 different digits are 9,8,7,6,5

Therefore, the largest 5- digit number having 5 different digits is 98765

b) The largest 5- digit number having 2 in the hundreds place is 98276.

c) The largest 5- digit number having 1 in the thousands place is 91876.

Exercise 1

Provide solutions to the following real-life problems:

1. Form a number with 7 at the ones place, 2 at the hundred thousand place, 5 at the hundreds place, 9 at the tens place, 4 at the thousands place, and 6 at the ten-thousands place.
2. I am a number with 9 at the thousands place, 4 at the hundred thousand place, 7 at the ten million place, I at the one million place, 8 at the ten-thousands place, 3 at the hundreds thousands place, 6 at the ones place, and 2 at the tens place. What number am I?
3. I am a six-digit number. My first digit is 5 more than the last digit, but 2 less than my second digit. My second digit is the third multiple of 3, while my fourth digit is the second multiple of 3. My third digit is the quotient when the fourth digit is divided by my last digit. However, my fourth and fifth digits are consecutive numbers. What number am I?
4. Find the product of the place values of two 5s in the numeral 30526541.
5. Form a number with 4 at the millions place, 3 at the ones place, 0 at the tens place, 8 at the thousand place, 9 at the hundred thousand place, 7 at the ten-thousand place, and 2 at the hundreds place.
6. Write the smallest 4- digit number.
   a. Having 4 different digits.
   b. Having 8 in the hundreds place.
   c. Having 6 in the tens place.

LESSON 3

Real number system

Real numbers are a combination of rational and irrational numbers. All arithmetic operations can be performed on real numbers and can be represented in the number line. Real numbers can also be defined as the union of both rational and irrational numbers. They can be both positive and negative. Real numbers are denoted by the symbol "R". Real numbers include natural numbers, decimals, and fractions. Examples of real numbers include:

1. Rational numbers eg ($\frac{3}{5}$, $\frac{7}{2}$, 0.53, $0.\bar{3}$...).
2. Integers eg (...-3, -2, -1, 0, 1, 2, 3,...).
3. Whole numbers eg (0, 1, 2, 3, 4,...).
4. Natural numbers eg (1, 2, 3, 4,...).
5. Irrational numbers eg $\sqrt{3}$,$\sqrt{5}$,π,0.10100110...

Set of Real Numbers

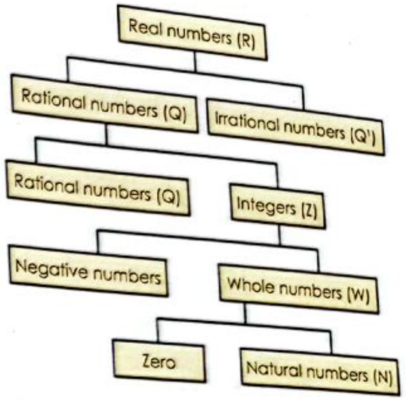
The set of real numbers consists of different groups. They include natural numbers, whole numbers, integers, rational numbers, and irrational numbers. Let us consider each of the classifications.

1. Natural numbers; contain all counting numbers which start from 1. Natural numbers are denoted by "N". Examples of natural numbers are (1, 2, 3,4,......).
2. Whole numbers: they include both zero and all-natural numbers. Whole numbers are denoted by "W". Examples include (0,1, 2, 3, 4,....).
3. Integers: They include all the whole numbers and the negatives of all the natural numbers. Integers are denoted by "Z". Examples include (...-3, -2, -1, 0, 1, 2, 3,....).
4. Rational number: These are numbers that can be written in the form $\frac{p}{q}$ where $q \neq 0$. Rational numbers are denoted by "Q" Examples include: $\frac{1}{2}$, $\frac{3}{2}$, $1\frac{1}{2}$, 0.5, $0.\overline{33}$.
5. Irrational numbers: These are numbers that cannot be written in the form $\frac{p}{q}$ Irrational numbers are denoted by "Q^1" Examples include the square root of each prime number, non-terminating decimals such as 0.1230125...

Differences between Rational and Irrational Numbers

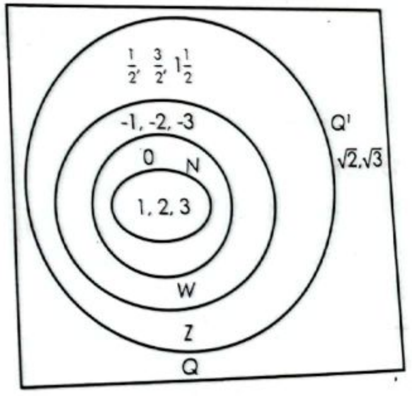
	Rational numbers	Irrational numbers
1	Denoted by Q	Denoted by Q'
2	Its decimal terminates or repeats	Its decimal does not terminate or repeat
3	The square root is a positive integer which is a rational	The square root of any positive integer that is not a perfect square is an irrational number
4	It can be written in the form $\frac{p}{q}$, $q \neq 0$	It cannot be written in the forum $\frac{p}{q}$

Real numbers chart



Real numbers can be represented on the set diagram to show the relationship among the real numbers namely; irrational numbers (Q^1), rational numbers (Q), integers (Z), whole numbers (W), and natural numbers (N).

Sets diagram



From the set diagram above,

N□W, W□Z, Z□Q

N ∩ W = N, W ∩ Z = W, Z ∩ Q = Z

N ∪ W = W, W ∪ Z = Z, Z ∪ Q = Q

Where □ is a subset of.

∩ is the intersection between sets.

∪ is the union of sets.

Exercise 1

For each of the following numbers, state whether it is a rational or irrational number.

a) $1.33\dot{3}$ h) -6.2222.....
b) $\sqrt{2}$ i) $\frac{4}{5}$
c) $\sqrt{9}$ j) $30.\dot{2}\dot{3}$
d) 5.123412341234... k) 0.4
e) 7.25 l) 0.09090091..
f) $3\sqrt{5}$ m) $\sqrt{7}$
g) $\sqrt{2} \times \sqrt{3}$ n) π

Representing real numbers on sets diagram

In this section, we shall represent real numbers on a set diagram. Let us consider the following worked examples.

Example 1

a) Write the factors of 6 and 12 and represent them on a Venn diagram.

b) From the diagram, find the intersection of the two sets.

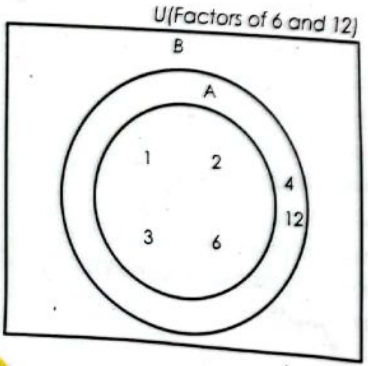
c) Find the union of two sets.

Solution

A= Factors of 6 = {1, 2, 3, 6}

B= Factors of 12 = {1, 2, 3, 4, 6, 12}

a)



b) From the diagram, AB = A∩B= {1,2,3,6}.

c) A∪B = {1, 2, 3, 4, 6, 12}.

Example 2

a) Write the factors of 15 and 18 and represent them on a Venn diagram.

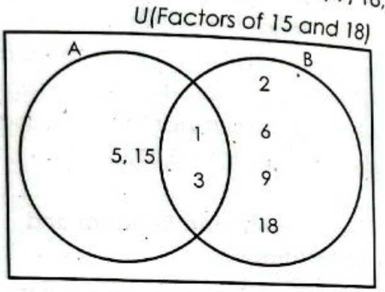
b) From the Venn diagram, find the intersection of the two sets.

c) The union of the two sets.

Solution

a) A = Factors of 15 = {1, 3, 5, 15}.

B = Factors of 18 = {1, 2, 3, 6, 9, 18,}.



b) From the diagram, A ∩ B = {1, 3}

c) A ∪ B = {1, 2, 3, 5, 6, 9, 15, 18}.

Exercise 2

1. Write the factors of 22 and 24 and represent them on a Venn diagram. Hence, find the intersection and union of the two sets.

2. Write the factors of the following pair of numbers and represent them on a Venn diagram:

a) 4 and 6 h) 12 and 15
b) 6 and 8 i) 16 and 18
c) 8 and 10 j) 18 and 20
d) 10 and 12 k) 22 and 26
e) 10 and 11 l) 10 and 20
f) 12 and 14 m) 24 and 26
g) 14 and 15 n) 26 and 28

3. a) . List the numbers of each of the sets A = {whole numbers from 20 to 30} and B = {Factors of 63}

b). List the members of 9i) A∩B (iii) A∪B.

4. Given that

Q = {Rational Numbers}

Q'= {Irrational Numbers}

Z = {Integers}

W = {Whole Numbers}

N= {Natural Numbers}

Represent the above information on a Venn diagram

Properties of Real Numbers

The five main properties of real numbers are

1. Commutative property.
2. Associative property.
3. Distributive property.
4. Identity property.
5. Closure property.

Commutative property

Given that p and q be real numbers.

The commutative property holds if $p + q = q + p$ for addition and p.q = q.p for multiplication.

Addition: $p + q. = q + p$

for example, 7+3 = 3+7, 4+2 = 2+4, 5+3 = 3+5

Multiplication: $p.q = q.p$.

For example,

7 x 3 = 3 x 7, 4 x 2 = 2 x 4,
5 x 3 = 3 x 5

Associative property

If P, q, and r are three different real numbers. The associative property holds if: $p + (q + r) = (p + q) + r$ for addition, and $p \times (q \times r) = (p \times q) \times r$ for multiplication.

Addition: The general form will be $P + (q + r) = (p + q) + r$. For example, 3 + (4 + 5) = (3 + 4) + 5

Multiplication: The general form will be $P \times (q \times r) = (p \times q) \times r$. for example, 3 x (4 x 5) = (3 x 4) x 5

Distributive property

Let p, q, and r be three different real numbers. The distributive property is represented as $p \times (q + r) = (p \times q) + (p \times r)$.

For example, 3 x (4 + 5) = (3 x 4) + (3 x 5). Therefore, multiplication is distributive over addition.

For example, 2(3 + 4) = 2 x 3 + 2 x 4
2(7) = 6 + 8
14 = 14

That is, both sides will produce 14

Identify property

There are additive and multiplicative identities.

For addition: m+0 = m, 3+0 = 3, 4+0 = 4.

Therefore, 0 is the additive identity. Thus, any number plus the additive identity yields that same number.

For multiplication; m x 1 = m, 1 x m = m, 3 x 1 = 3, 4 x 1 = 4, 7 x 1 = 1. (1 is the identity element for multiplication). Thus, the identity element for multiplication multiplying any number produces that same number.

Take note

1. Subtraction and division of rational numbers are not commutative.
2. Subtraction and division of rational numbers are not associative.
3. Subtraction and division of rational numbers are not distributive.

Closure property

If an operation is performed on elements of a given set and the result belongs to the given set, then that set is closed under the operation. Note the following:

1. The sets of natural numbers (N), whole numbers (W), integers (Z), and rational numbers (Q) are closed under addition and multiplication. For example, the sum of any two natural numbers yields another natural number.
2. Only the sets of integers (Z) and rational numbers (Q) are closed under subtraction. For example, the difference between any two rational numbers yields another rational number.

   However, the sets of natural numbers (N) and whole numbers (W) are not closed under subtraction.
3. If the operation + is defined over the $T = \{1, 2, 3, 4\}$ then $2 + 3 = 5$ since 5 does not belong to the set T, then the set T is not closed under addition.

Example 1:

List five rational numbers between $\frac{1}{2}$ and $\frac{3}{5}$

Solution

Make the denominators the same for both given rational numbers

The Least Common Multiple (LCM) for 2 and 5 is 10.

The equivalent fraction for

$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

The equivalent fraction from $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ Since we need 5 rational numbers between $\frac{1}{2} = \frac{5}{10}$ and $\frac{3}{5} = \frac{6}{10}$ multiply the numerator and denominator of each rational number by 6. $\frac{5 \times 6}{10 \times 6} = \frac{30}{60}$ and $\frac{6 \times 6}{10 \times 6} = \frac{36}{60}$

Five rational numbers between $\frac{1}{2} = \frac{30}{60}$

And $\frac{3}{5} = \frac{36}{60}$ are $\frac{31}{60}, \frac{32}{60}, \frac{33}{60}, \frac{34}{60}, \frac{35}{60},$

Example 2

Write the decimal equivalent of

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{2}$

Solution

a) $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$

b) $\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 0.4$

c) $\frac{5}{2} = \frac{5 \times 5}{2 \times 5} = \frac{25}{10} = 2.5$

Example 3

What should be multiplied by 2.35 to get the answer 1

Solution

$2.35 = \frac{235}{100}$

Now, multiply $\frac{235}{100}$ by its reciprocal

Thus, $\frac{235}{100} \times \frac{100}{235}$

Therefore, if we multiply 2.35 by $\frac{235}{100}$, the answer is 1

Example 4:

The operation * is defined as $p*q = p + q + pq$

a) Evaluate (i) 2 * 3 (ii) 3 * 2
b) Compare your answers in (i) and (ii)
c) What property is that?

Solution

Given $p * q = p + q + pq$

a) (i) $2 * 3 = 2 + 3 + 2 \times 3$
$= 5 + 6$
$= 11$

ii) $3 * 2 = 3 + 2 + 3 \times 2$
$= 5 + 6$
$= 11$

b) $2 * 3 = 3 * 2$

c) This is communicative property.

Example 5:

The operation Δ is defined over the set of real numbers $m \Delta n = m + n + mn$.

a. Evaluate (i) 2Δ (3Δ4) ii) (2Δ3)Δ4
b. Compare your answers in (i) and (ii).
c. What property is that?

Solution

a. Given $m \Delta n = m + n + mn$.

i) 2Δ (3Δ4)

Bracket first: $(3 \Delta 4) = 3 + 4 + 3 \times 4$
$= 7 + 12$
$= 19$

$2 \Delta (3 \Delta 4) = 2 \Delta 19 = 2 + 19 + 2 \times 19$
$= 21 + 38$
$= 59$

ii) $(2 \Delta 3) \Delta 4 =$

Bracket first: $2 \Delta 3 = 2 + 3 + 2 \times 3$
$= 5+6$
$= 11$

$(2 \Delta 3) \Delta 4) = 11 \Delta 4 = 11 + 4 + 11 \times 4$
$= 15 + 44$
$= 59$

b. $2 \Delta (3 \Delta 4) = (2 \Delta 3) \Delta 4$

c. This is associative property

Example 6:

If $(3.14 \times 18) \times 17.5 = 3.14 \times (3a \times 17.5)$
Find the value of "a"

Solution

By comparison $18 = 3a$

Divide both sides by 3

$\frac{18}{3} = \frac{3a}{3}$

$6 = a$

Therefore, the value of a is 6

Example 7:

Simplify

a. $2 \times (3\frac{1}{3} + 1\frac{1}{6})$
b. $2 \times 3\frac{1}{3} + 2 \times 1\frac{1}{6}$

c. Compare the results in (a) and (b) above.

d. What property is that?

Solution

a. $2 \times (3\frac{1}{3} + 1\frac{1}{6}) = 2 \times (\frac{10}{3} + \frac{7}{6})$

$= 2 \times (\frac{20+7}{6})$ bracket first

$= 2 \times (\frac{27}{6})$

$= 9$

b. $2 \times 3\frac{1}{3} + 2 + 1\frac{1}{6} = 2 \times \frac{10}{3} + 2 \times \frac{7}{6}$

$= \frac{20}{3} + \frac{7}{3}$

$= \frac{72}{3}$

$= 9$

c. $2 \times 3\frac{1}{3} + 2 \times 1\frac{1}{6} = 2 \times \frac{10}{3} + 2 \times \frac{7}{6}$

d. This is the distributive property.

Exercise 3

1. Find the set of prime factors of 12.
2. Find the value of $\sqrt{6\frac{1}{4}}$
3. Find the least whole number which must be added to 207 to make it divisible by 17
   A. 0 B. 3 C. 13 D. 14
4. List the set of factors of 12.
5. What property of arithmetic operation is illustrated by $a(b + c) = ab + ac$?
   A. Addition B. Associative
   C. Commutative D. Distributive
6. What property is illustrated by the statement $a + (b + c) = (a + b) + C$?
7. If the product of 6287 x 543 = 3413841, what is the value of 628.7 x 5.43?
   A. 3,413841 B. 341384.1
   C. 34138.1 D. 3413.841
8. State the property used in the statement $p(q + r) = (pq) + (pr)$.
9. If (23 x 82) x 79 = 148994, find the exact value of (2.3 x 82) x 7.9
10. What property of addition is defined by $(a + b) + c = a + (b + c)$?
11. State the property used in the operation $a + b = b + a$.
12. Which of the following numbers is the next prime number greater than 23?
    A. 17 B. 24 C. 25 D. 29
13. If x is an integer list the members of the set $\{2 \leq x < 10\}$
14. Write down all the integers within the interval $2 < y \leq 27$.
15. If p = {x: x is an even number greater than two and less or equal to twelve}. List the members of P.

Exercise 4

1. Which of the following statements is/are true?
   a) 3 × 5 = 5 × 3
   b) 3 - 5 = 5 - 3
   c) 3 + 5 = 5 + 3
   d) 3 ÷ 5 = 5 ÷ 3
   e) 3 + (4 + 5) = (3 + 4) + 5
   f) 3 × (4 × 5) = (3 × 4) × 5
   g) 3 - (4 - 5) = (3 - 4) – 5
   h) 3 × (4 + 5) 3 × 4 + 3 × 5

2. The operation * is defined as m*n $= m - n$
   a. Evaluate (i) 2 *3 (ii) 3*2
   b. Compare your answers in (i) and (ii).
   c. What is your conclusion on your answers in (i) and (ii)?
3. The operation Δ is defined as p*q $= p + q - pq$
   a. Evaluate (i) 2 Δ (4 Δ 5) (ii) (2 Δ 4) Δ 5
   b. Compare your answers in (i) and (ii).
   c. What property is that?
4. Use the distributive proper, and find the value of m in the following:
   a. $2(m + 5) = (2 \times m) + (2 \times 5)$
   b. $7(3 + m) = (7 \times 3) + (7 \times m)$
   c. $5(4 + m) = (5 \times 4) + (5 \times m)$
   d. $3(5 + m) = (3 \times 5) + (3 \times m)$
5. The operation * is defined on the set of real numbers by a*b $= a + b + 5$
   a. Evaluate 2 * (3 + 4)
   b. (2 * 3) + (2 * 4)

Rational numbers on a number line

Positive rational numbers are always represented on the right side of the zero on the number line. While negative rational numbers are always represented on the left side of zero on the number line. The representation of rational numbers on the number line depends upon the type of rational fraction to be represented on the number line. Below are some of the types of rational numbers and ways to represent them on the number line.

1. Proper fraction

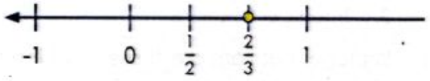
Proper fractions are those in which the numerator is less than the denominator. Such a fraction exists between only zero and one. Proper fractions are greater than zero and less than one. Let us consider the following examples:

Example 1:

Represent $\frac{2}{3}$ on the number line.

Solution

Since the given rational number is greater than zero but less than one, it will be represented on the right side of zero on the number line. So, first of all, we need to divide the number line between zero and one into three equal parts and the second part of the three parts will be the representation of $\frac{2}{3}$ on the number line. It can be represented as shown below.



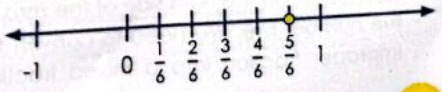
Therefore, $\frac{2}{3}$ is represented on the above number line

Example 2:

Represent $\frac{5}{6}$ on the number line.

Solution

The given $\frac{5}{6}$ is positive and that too is a proper fraction, so it will lie at the right side of the zero and will be less than one. To do so, we first divide the number line between zero and one into six equal parts. The $\frac{5}{6}$ with be the fifth part of six equal parts. Let us represent this on the number below.



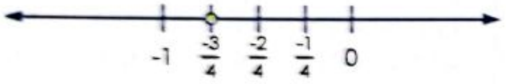
Therefore $\frac{5}{6}$ is represented on the above number line.

Example 3:

Represent $\frac{-3}{4}$ on the number line.

Solution

The given fraction $\frac{-3}{4}$ is negative but a proper fraction. So, it will be smaller than zero but greater than -1. Therefore, the fraction will be between zero and a negative one. To represent this, we will divide the number line between zero and negative one into four equal parts and the third part of the four parts will be $\frac{-3}{4}$ this can be represented as seen below.



Therefore the $\frac{-3}{4}$, is represented on the above number line.

2. Improper fractions

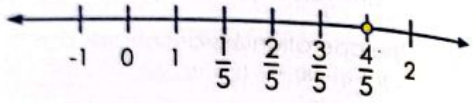
Improper fractions are those in which the numerator of the fraction is greater than the denominator. Since the numerator is greater than the denominator, the number will be greater than one. To represent such rational fractions on the number line, first convert the improper fraction into the mixed fraction to know between which integers will the fraction lie. Let us consider the following examples:

Example 4:

Represent $\frac{9}{5}$ on the number line.

Solution

The given fraction is improper and is positive so, it will lie on the right side of the zero on the number line. We need to convert the improper fraction into a mixed fraction. Therefore $\frac{9}{5} = 1\frac{4}{5}$, so, the fraction will be between 1 and 2 at $\frac{4}{5}$ point. To represent it we will divide the number line between 1 and 2 into five equal parts and the fourth part of the five parts will be the required rational fraction. This is shown below.



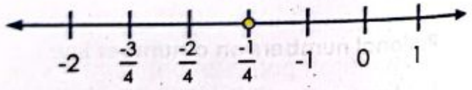
Therefore, $\frac{9}{5} = 1\frac{4}{5}$ is represented on the above number line.

Example 5:

Represent $\frac{-5}{4}$ on the number line.

Solution

The given fraction is improper and is negative so, it will lie on the left side of the zero on the number line. We need to convert the improper fraction into a mixed fraction. Therefore $\frac{-5}{4} = -1\frac{1}{4}$, so, the fraction will be between -1 and -2. To represent it we will divide the number line between -1 and -2 into four equal parts and the first part of the four parts will be the required rational fraction. This is shown below



Therefore, $\frac{-5}{4} = -1\frac{1}{4}$ is represented on the above number line.

Exercise 5

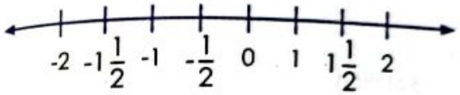
Represent the following rational numbers on the number line

a) $\frac{3}{5}$ b) $\frac{-4}{7}$ c) $\frac{5}{7}$ d) $\frac{-7}{8}$ e) $\frac{3}{4}$

f) $\frac{5}{3}$ g) $\frac{-7}{4}$ h) $\frac{7}{5}$ i) $\frac{8}{7}$ j) $\frac{-4}{3}$

Comparing and ordering rational numbers

We can compare and order rational numbers using the number line. First represent the given rational numbers on the same number line to determine which rational number is the greatest. A number on the right side of the number on the number line is greater than the number on the left side of it.



On the number line above $\frac{1}{2}1$ is greater than $-1\frac{1}{2}$ because $1\frac{1}{2}$ is at the right side of $-1\frac{1}{2}$ on the number line. $-1\frac{1}{2}$

Example 1.

Use a number line to determine which of the following pairs of numbers is greater:

a. $3\frac{1}{2}$ and $4\frac{1}{2}$

b. $-4\frac{1}{2}$ and $-1\frac{1}{2}$

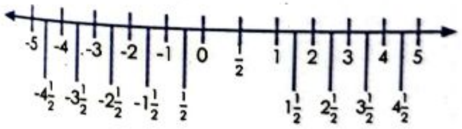
c. $1\frac{1}{2}$ and $-2\frac{1}{2}$

d) $2\frac{1}{2}$ and $-3\frac{1}{2}$

e) $2\frac{1}{2}$ and $-2\frac{1}{2}$

f) $1\frac{1}{2}$ and $-1\frac{1}{2}$

Solution



From the number line above.

a) $4\frac{1}{2} > 3\frac{1}{2}$

b) $-4\frac{1}{2} < -1\frac{1}{2}$

c) $1\frac{1}{2} > -2\frac{1}{2}$

d) $2\frac{1}{2} > -3\frac{1}{2}$

e) $2\frac{1}{2} > -2\frac{1}{2}$

f) $1\frac{1}{2} > -1\frac{1}{2}$

Exercise 6

Determine which of the following pairs of a rational number is greater using > or <

a) $\frac{1}{4}$ and $\frac{-2}{5}$

b) $-6\frac{1}{3}$ and $-5\frac{1}{2}$

c) $5\frac{3}{4}$ and $5\frac{1}{4}$

d) -4 and $-3\frac{1}{4}$

e) $-2\frac{2}{3}$ and -3

f) -5 and $-4\frac{1}{3}$

g) $-2\frac{1}{3}$ and $-3\frac{1}{5}$

h) $-3\frac{1}{4}$ and $-3\frac{3}{4}$

i) $\frac{-2}{3}$ and $-4\frac{1}{3}$

j) $\frac{1}{5}$ and 1

Exercise 7

1. Arrange the following fractions $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$ in ascending order.
2. Arrange the following fractions from the highest to the lowest $\frac{3}{4}, -9, \frac{3}{5}$ and 0
3. Which of the following fractions is the greatest? $\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}$
4. Arrange the following rational numbers from the lowest to the highest: $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$
5. Which of the following fractions $\frac{13}{20}, \frac{3}{5}, \frac{3}{4}$, and $\frac{7}{10}$ is the greatest?
6. Arrange the following fractions in ascending order $4\frac{1}{4}, 4\frac{1}{2}, 4\frac{1}{3}$
7. Arrange the following fractions from the highest to the lowest $\frac{5}{6}, \frac{4}{5}$, and $\frac{4}{7}$
8. Arrange the following fractions in descending order of magnitude: 0.32, $\frac{2}{5}$, 27%, $\frac{1}{3}$
9. Arrange the following fractions in ascending order: $\frac{7}{12}, \frac{3}{5}, \frac{7}{15}, \frac{3}{4}$
10. Arrange the following fractions in ascending order of magnitude: $\frac{2}{3}, \frac{5}{7}, \frac{2}{5}, \frac{1}{2}$

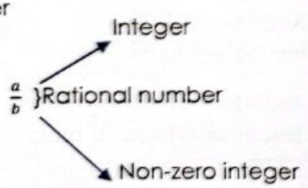
11. Arrange the following in ascending order of magnitude: $\frac{2}{3}, \frac{5}{12}$, and $\frac{3}{4}$.
12. Arrange the following fractions in ascending order of magnitude:
    - a) $\frac{5}{8}, \frac{11}{20}, \frac{7}{10}$
    - b) $\frac{7}{20}, \frac{7}{25}, \frac{37}{100}, \frac{1}{4}$
    - c) $\frac{2}{3}, \frac{4}{9}, \frac{3}{7}$

Operations on Real Numbers

Operations on rational numbers are carried out, in the same way, the arithmetic operation like subs traction, addition, multiplication, and division on integers and fractions. Rational numbers are expressed in the form $\frac{a}{b}$ $b \neq 0$. Rational numbers are not called fractions because fractions include only positive numbers while rational numbers include both positive and negative numbers. Fractions are a subset of rational numbers. In this section, we shall consider addition, subtraction, multiplication, and division of rational numbers. Examples of rational numbers include

$\frac{1}{2}, \frac{1}{3}$, 0.5, $\frac{-1}{4}$, 0.6, etc.

Let us consider the following rational number



Addition of rational numbers

Adding rational numbers can be done in the same way as adding fractions. There are two cases related to the addition of rational numbers.

1. Adding rational numbers with like denominators
2. Adding rational numbers with different denominators

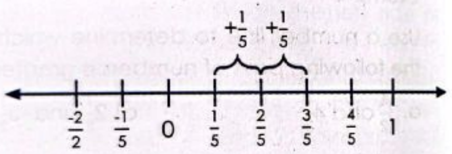
Two or more rational numbers with like denominators can be added by adding all the numerators and write the common denominator. For example,

Example 1.

Solve $\frac{1}{5} + \frac{2}{5}$

Solution

This can be done using the number line. We will start from $\frac{1}{5}$. Then move 2 steps towards the right side as we are adding $\frac{2}{5}$. We finally reach $\frac{3}{5}$. Thus $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$



Example 2:

solve the following:

a. $\frac{1}{4} + \frac{2}{4}$

b. $\frac{3}{7} + \frac{2}{7}$

c. $\frac{3}{10} + \frac{5}{10}$

d. $\frac{5}{12} + \frac{6}{12}$

Solution

a. $\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$

b. $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

c. $\frac{3}{10} + \frac{5}{10} = \frac{3+5}{10} = \frac{8}{10} = \frac{4}{5}$

d. $\frac{5}{12} + \frac{6}{12} = \frac{5+6}{12} = \frac{11}{12}$

when rational numbers have different denominators, we first make their denominators equivalent using the LCM of their denominators. Let us consider the following examples.

a. $\frac{1}{3} + \frac{3}{5}$

b. $\frac{-2}{5} + \frac{1}{10}$

c. $\frac{2}{3} + \frac{1}{6}$

d. $\frac{1}{4} + \frac{3}{8}$

Solution

a) The Least Common Multiple (LCM) of 3 and 5 is 15.

Equivalent rational numbers with the common denominator

$\frac{1}{3} = \frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$

$\frac{3}{5} = \frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$

Therefore $\frac{1}{3} + \frac{3}{5} = \frac{5}{15} + \frac{9}{15}$

$= \frac{5+9}{15}$

$= \frac{14}{15}$

b) $\frac{-2}{5} + \frac{1}{10}$

The LCM of 5 and 10 is 10

Find equivalent rational numbers with the common denominator

$\frac{-2}{5} = \frac{-2}{5} \times \frac{2}{2} = \frac{-4}{10}$

$\frac{1}{10} = \frac{1}{10} \times \frac{1}{1} = \frac{1}{10}$

Therefore, $\frac{-2}{5} + \frac{1}{10} = \frac{-4}{10} + \frac{1}{10}$

c) $\frac{2}{3} + \frac{1}{6}$

LCM of 3 and 6 is 6.

Find an equivalent rational number with the common denominator

$\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

$\frac{1}{6} = \frac{1}{6} \times \frac{1}{1} = \frac{1}{6}$

Therefore, $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6}$

$= \frac{4+1}{6}$

$= \frac{5}{6}$

d) $\frac{1}{4} + \frac{3}{8}$

LCM of 4 and 8 is 8

Find equivalent rational numbers with the common denominator

$\frac{1}{4} = \frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$

$\frac{3}{8} = \frac{3}{8} \times \frac{1}{1} = \frac{2}{8}$

Therefore, $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8}$

$= \frac{2+3}{8}$

$= \frac{5}{8}$

Subtraction of Rational Numbers

The process of subtraction of rational numbers is the same as that of addition. There are two cases related to the subtraction of rational numbers.

1. Subtracting rational numbers with like denominators.
2. Subtracting rational numbers with different denominators.

Two or more rational numbers with like denominators can be subtracted by subtracting the numerators and maintaining the denominators. Let us consider the following worked examples:

Example 1:

Solve the following:

a) $\frac{13}{7} - \frac{5}{7}$ b) $\frac{-4}{5} - \frac{3}{5}$

c) $\frac{-8}{13} - \frac{4}{13}$ d) $\frac{2}{19} - \frac{7}{19}$

Solution

a. $\frac{13}{7} - \frac{5}{7} = \frac{13-5}{7}$

$= \frac{8}{7}$

$= 1\frac{1}{7}$

b. $\frac{-4}{5} - \frac{3}{5} = \frac{-4-3}{5}$

$= \frac{-7}{5}$

$= -1\frac{2}{5}$

c. $\frac{-8}{13} - \frac{4}{13} = \frac{-8-4}{13}$

$= \frac{-12}{13}$

d. $\frac{2}{19} - \frac{7}{19} = \frac{2-7}{19}$

$= -\frac{5}{19}$

To subtract rational numbers with different denominators, find the LCM of the denominators and find their equivalent rational numbers. Let us consider the following work examples:

Example 2:

Solve the following:

a. $\frac{1}{2x} - \frac{1}{3x}$ b) $\frac{5}{6} - \frac{3}{4}$ c) $\frac{-5}{7} - \frac{3}{8}$

Solution

$\frac{1}{2x} - \frac{1}{3x}$

The LCM of $2x$ and $3x$ is $6x$. find the equivalent rational number with the common denominator

a. $\frac{1}{2x} = \frac{1}{2x} \times \frac{3}{3} = \frac{3}{6x}$

$\frac{1}{3x} = \frac{1}{3x} \times \frac{2}{2} = \frac{2}{6x}$

Therefore, $\frac{1}{2x} - \frac{1}{3x} = \frac{3}{6x} - \frac{2}{6x}$

$= \frac{3-2}{6x}$

$= \frac{1}{6x}$

b. $\frac{5}{6} - \frac{3}{4}$

The LCM of 6 and 4 is 12. Find equivalent rational numbers with the common denominator.

$\frac{5}{6} = \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$

$\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

Therefore $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12}$

$= \frac{10-9}{12}$

$= \frac{1}{12}$

c. $\frac{-5}{7} - \frac{3}{8}$

The LCM of 5 and 8 is 56. Find an equivalent rational number with the common denominator

$\frac{-5}{7} = \frac{-5}{7} \times \frac{8}{8} = \frac{-40}{56}$

$\frac{-3}{8} = \frac{-3}{8} \times \frac{7}{7} = \frac{21}{56}$

Therefore $\frac{-5}{7} - \frac{3}{8} = \frac{-40}{56} - \frac{21}{56}$

$= \frac{-61}{56}$

$= -1\frac{5}{56}$

Take note

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that b and d do not have a common factor other than 1 then,

$\frac{a}{b} - \frac{c}{d} = (a \times d - c \times b) \div b \times d$

Exercise 8

Simplify the following

1. $7\frac{1}{6} + 3\frac{2}{5} - 6\frac{2}{3}$
2. $3\frac{1}{2} + 5\frac{2}{3} - 4\frac{1}{2}$
3. $\frac{3}{5} + \frac{1}{2}$
4. $-4\frac{1}{3} + 2\frac{1}{4}$
5. $-3\frac{3}{4} + \frac{7}{8}$.
6. $\frac{-5}{7} - \frac{3}{7}$
7. Subtract 9 from $\frac{4}{5}$.
8. Solve $\frac{-3}{4} - \frac{5}{6}$
9. Simply $\frac{3}{-15} - \frac{7}{-12}$
10. Simplify $\frac{11}{-18} - \frac{5}{12}$

Hint. write each one of the rational numbers with a positive denominator.

Multiplication of Rational Numbers

The multiplication of rational numbers is similar to how we multiply fractions. Consider the following steps in multiplying two or more rational numbers.

Step 1: multiply the numerators.

Step 2: Multiply the denominators.

Step 3: Reduce the resulting number to its lowest term.

Example 1:

Simplify $\frac{-2}{3} \times \frac{-3}{4}$

Solution

$\frac{-2}{3} \times \frac{-3}{4}$

Steps1 and 2

Multiply the numerators and also multiply the denominators

$\frac{-2}{3} \times \frac{-3}{4} = \frac{-2 \times -3}{3 \times 4}$

$= \frac{6}{12}$

$= \frac{1}{2}$

That is reducing the resulting number to its lowest term.

Example 2:

Sara uses $\frac{3}{5}$ of the flour if she has to bake a full cake. How much flour will Sara use to bake $\frac{1}{4}$ the portion of the cake?

Solution

Total flour to bake a full cake $= \frac{3}{5}$.

Using operations on rational numbers, the amount of flour used to bake $\frac{1}{4}$ the portion of the cake =

$\frac{3}{5} \times \frac{1}{4} = \frac{3 \times 1}{5 \times 4}$

$= \frac{3}{20}$

Therefore, Sara would have to use $\frac{3}{20}$ of the flour.

Example 3:

Simplify the following:

a. $\frac{9}{6} \times \frac{27}{3}$ b. $\frac{-5}{15} \times \frac{21}{10}$

c. $\frac{-4}{9} \times \frac{-3}{2}$ d. $\frac{10}{20} \times (\frac{-5}{30})$

Solution

a. $\frac{9}{6} \times \frac{27}{3} = \frac{1}{2} \times \frac{27}{1}$

$$\frac{9}{6} \times \frac{27}{3} = \frac{1}{2} \times \frac{27}{1}$$
$$= \frac{27}{2}$$
$$= 13\frac{1}{2}$$

b. $\frac{-5}{15} \times \frac{21}{10} = \frac{-1}{3} \times \frac{21}{10}$
$$= \frac{-21}{30}$$

c. $\frac{-4}{9} \times \frac{-3}{2} = \frac{-2}{3} \times \frac{-1}{1}$
$$= \frac{2}{3}$$

d. $\frac{10}{20} \times (\frac{-5}{30}) = \frac{1}{2} \times \frac{-1}{6}$
$$= \frac{-1}{12}$$

Example 4

Simplify the following:

a. $\frac{2}{7} \times \frac{4}{5}$

b. $\frac{-7}{6} \times 2$

b) $\frac{5}{9} \times \frac{-2}{3}$

d) $\frac{-3}{5} \times \frac{15}{6}$

Solution

a. $\frac{2}{7} \times \frac{4}{5} = \frac{2 \times 4}{7 \times 5}$
$$= \frac{8}{35}$$

b. $\frac{5}{9} \times \frac{-2}{3} = \frac{5}{9} \times \frac{-2}{3}$
$$= \frac{-10}{27}$$

c. $\frac{-7}{6} \times 2 = \frac{-7}{6} \times \frac{2}{1}$
$$= \frac{-14}{6}$$
$$= -2\frac{2}{6} \text{ or } -2\frac{1}{3}$$

d. $\frac{-3}{5} \times \frac{15}{6} = \frac{-1}{1} \times \frac{3}{2}$
$$= \frac{-3}{2}$$
$$= -1\frac{1}{2}$$

Exercise 9

Simplify the following:

a. $\frac{13}{6} \times \frac{-18}{91}$

b. $\frac{2}{7} \times \frac{3}{5}$

c. $\frac{-7}{5} \times 15$

d. $\frac{15}{6} \times \frac{28}{55}$

e. $\frac{13}{6} \times \frac{18}{91}$

f. $\frac{2}{5} \times \frac{3}{7}$

g. $\frac{-3}{4} \times \frac{3}{9}$

h. $\frac{-3}{7} \times \frac{21}{5}$

i. $\frac{-11}{6} \times \frac{28}{55}$

j. $\frac{-3}{15} \times \frac{45}{12}$

Division of Rational Numbers

Dividing a number by a rational number is done by multiplying the number by the reciprocal of the rational number. In more formal terms:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Let us consider the following examples:

Example 1:

Divide the following rational numbers giving your answer in the simplest form.

a. $\frac{1}{3} \div \frac{1}{4}$

b. $\frac{3}{7} \div \frac{3}{2}$

c. $\frac{-m}{3} \div \frac{1}{6y}$

d. $\frac{8}{2x} \div \frac{y}{x}$

Solution

a. $\frac{1}{3} \div \frac{1}{4}$

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$

$$\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1}$$
$$= \frac{1}{3} \times \frac{4}{1}$$
$$= \frac{4}{3}$$
$$= 1\frac{1}{3}$$

b. $\frac{3}{7} \div \frac{3}{2}$

Solution

The reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$

Therefore $= \frac{3}{7} \div \frac{3}{2} = \frac{3}{7} \times \frac{2}{3}$
$$= \frac{1}{7} \times \frac{2}{1}$$
$$= \frac{2}{7}$$

c. $\frac{-m}{3} \div \frac{1}{6y}$

Solution

The reciprocal of $\frac{1}{6y}$ is $\frac{6y}{1}$

Therefore, $\frac{-m}{3} \div \frac{1}{6y} = \frac{-m}{3} \times \frac{6y}{1}$
$$= \frac{-m}{1} \times \frac{2y}{1}$$
$$= \frac{-2my}{1}$$
$$= -2my$$

d. $\frac{8}{2x} \div \frac{y}{x}$

Solution

The reciprocal of $\frac{y}{x}$ is $\frac{x}{y}$

Therefore, $\frac{8}{2x} \div \frac{x}{y} = \frac{8}{2x} \times \frac{x}{y}$
$$= \frac{4}{1} \times \frac{1}{y}$$
$$= \frac{4}{y}$$

Example 2:

Simplify the following:

a. $2\frac{1}{2} \div \frac{3}{4}$

b) $\frac{3}{10} \div \frac{7}{15}$

c $\frac{3x}{5} \div \frac{9}{5}$

Solution

a. $2\frac{1}{2} \div \frac{3}{4}$

they go through a

Change mixed numbers to improper fractions

$\frac{5}{2} \div \frac{3}{4}$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

Therefore $\frac{5}{2} \div \frac{3}{4} = \frac{5}{2} \times \frac{4}{3}$
$$= \frac{5}{2} \times \frac{4}{3}$$
$$= \frac{5}{1} \times \frac{2}{3}$$
$$= \frac{10}{3}$$
$$= 3\frac{1}{3}$$

b. $\frac{3}{10} \div \frac{7}{15}$

Solution

The reciprocal of $\frac{7}{15}$ is $\frac{15}{7}$

Therefore $\frac{3}{10} \div \frac{7}{15} = \frac{3}{10} \times \frac{15}{7}$
$$= \frac{3}{2} \times \frac{3}{7}$$
$$= \frac{9}{14}$$

c. $\frac{3x}{5} \div \frac{9}{5}$

Solution

The reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$

Therefore, $\frac{3x}{5} \div \frac{9}{5} = \frac{3x}{5} \times \frac{5}{9}$
$$= \frac{x}{1} \times \frac{1}{3}$$
$$= \frac{x}{3}$$

Exercise 10

Simplify the following:

a. $\frac{1}{4} \div \frac{2}{5}$

b. $\frac{11}{5} \div \frac{7}{6}$

g) $\frac{7}{9} \div \frac{1}{2}$

h) $\frac{-y}{4} \div \frac{7}{2}$

c. $\frac{1}{3} \div \frac{2}{3}$

d. $\frac{8}{3} \div \frac{15}{16}$

e. $\frac{7}{15} \div \frac{4}{5}$

i) $-2\frac{1}{2} \div 3\frac{2}{6}$

j) $\frac{2}{3} \div \frac{3}{2}$

k) $\frac{5}{11} \div \frac{1}{2}$

Exercise 11

1. Simplify $\frac{1}{2}(1\frac{1}{2} + \frac{3}{4} \div \frac{1}{4})$.
2. Simplify $2 \times (3\frac{1}{3} + 1\frac{3}{4} \div \frac{1}{4})$
3. Simplify $(3\frac{1}{2} + 7) \div (4\frac{1}{3} - 3)$
4. Evaluate $(\frac{2}{3} - \frac{1}{4}) \div \frac{5}{6}$
5. Divide $(1\frac{1}{2} + \frac{1}{4})$ by $(1\frac{1}{2} - \frac{1}{4})$
6. Simplify $\frac{1}{2} - \frac{1}{4} + \frac{1}{8}$
7. Simplify $(\frac{2}{3} + 6\frac{3}{4}) \div (2\frac{4}{15} - 1\frac{2}{3})$
8. Simplify $1\frac{1}{2} + 2\frac{1}{4} - 3\frac{5}{8}$
9. Simplify $(2\frac{2}{4} \div (3\frac{3}{8} - 1\frac{1}{12})$
10. Simplify $\frac{1}{3}(2\frac{2}{3} + \frac{5}{6})$
11. Solve $2\frac{3}{4} \div (3\frac{3}{8} - 1\frac{1}{2})$
12. Evaluate $\frac{37}{100} \times \frac{7}{10}$
13. Simplify the following:
    a. $(\frac{2}{3} + 6\frac{3}{4}) \div (2\frac{4}{15} - 1\frac{2}{3})$
    b. $(\frac{2}{3} - \frac{1}{2}) \div \frac{1}{6}$
    c. $\frac{1}{2} - \frac{2}{3} + \frac{3}{4}$
    d. $14\frac{1}{3} - 2\frac{3}{8} + 5\frac{7}{12}$
    e. $7\frac{2}{3} - 4\frac{5}{6} + 2\frac{3}{8}$
    f. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27}$
    g. $\frac{4}{5} - \frac{1}{3} + \frac{2}{9}$
    h. $7\frac{1}{3} \times (\frac{1}{4} \div \frac{1}{2}) - \frac{1}{4}$

LESSON 4

Sets

Sets are a collection of well-defined objects or elements and it does not change from person to person. A set is represented by a capital letter. The number of elements in a finite set is called the cardinality of the set. Sets contain elements or members such as numbers, symbols, points in space, geometrical shapes, lines, and many more. The elements of the sets are enclosed in curly brackets and are separated by commas, for example, set A = {1, 3, 5, 7, 9}, which is the collection of the first five odd numbers. The symbol "ϵ" is used to donate that an element is contained in a set. In the above example 3ϵA. If an element is not a member of a given set, then it is denoted by the symbol ⋸. In the above example, there are five elements in set A. this is denoted as n(A)= 5.

Therefore, the cardinality of set "A" is 5.

Representation of Sets

Sets can be represented in different ways. They differ in the way in which the elements are listed. Let us consider the following.

a. Semantic form

b. Roster form

c. Set builder form

Semantic form

The semantic notation uses words to describe a statement to show what are the elements of a set. For example, the elements in set "A" are described as the list of the first five odd numbers.

Roster form

The most common form used to represent sets is the roster notation in which members of the set are listed, separated by commas, and enclosed in curly brackets. For example, set.

P= {2, 4, 6, 8, 10}. The order of the members of a set in a roster form is not important.

For example, the set of the first five even numbers can also be defined as {4, 2, 8, 6, 10}. Elements in the roster notation can be represented in the following ways.

a. Finite roster notation of sets:
set Q = {1, 2, 3, 4}

b. Infinite Roster notation of sets:
set R= {3, 6, 9, 12,....}

Set Builder Form

The set builder notation has a statement that describes the elements of a set. The set builder form uses a vertical bar in its representation, followed by a text describing the character of the elements of the set. For example, Y= {m | m is an odd number,m<19}. This statement indicates that all the elements of set Y are odd numbers that are less than 19. In some cases, ":" is used in replace of the vertical bar. "|".

Example 1:

Find the elements of the sets represented as follows and write the cardinality of each set.

a. Set P is the first 10 multiples of 5.

b. Set Q = {a, e, i, o, u}

c. Set C = {y/y are even numbers between 10 and 20}

Solution

a. P= {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}
These are the first 10 multiples of 5 since there are 10 elements in set P, then the cardinal number of n(P) = 10.

b) Q = {a, e, i, o, u}
Since there are 5 elements in set Q, then the cardinal number n(Q) = 5.

c) C = {12, 14, 16, 18}
These are the even numbers between 10 and 20

Since there are 4 elements in set C, then the cardinal number n(C) = 4.

Example 2:

Represent the given set-in set–builder notation form.

R= {1, 3, 5, 7, 9, 11, 13}

Solution

Given: R= {1, 3, 5, 7, 9, 11, 13}

Using sets notation, we can represent the given set R in the set-builder form as

R= {y/y is on odd number less than 16}

Exercise 1

List the elements of the following sets:

1. A is the set of vowels of the English alphabet.
2. B is the set of odd numbers less than or equal to 15.
3. C is the set of even numbers less than 14.
4. D is the multiple of 5 between 15 and 45.

5. E is the set of months with less than 30 days.

6. F is the set of prime numbers less than 15.

7. G is the set of months with 30 days.

8. H is the set of natural numbers between 10 and 20.

9. J is the set of integers greater than -2.

Exercise 2

Use the set-builder notation to describe the following sets:

1. P is the set of even numbers between 4 and 26.
2. Q is the set of squares.
3. R is the set of odd numbers less than 15.
4. S is the set of integers less than 5.
5. T is the set of multiples of 3 less than 21.
6. U is the set of factors of 20.
7. Write down the cardinality of the following sets:
   a. A = {1, 2, 3, 4, 5, 6, 7}.
   b. M = {a, b, c, d, e}.
   c. N = {x:x is an even number, $10 \leq x \geq 20$}.
   d. P = {3, 6, 9, 12, 15, 18}.

Types of sets

Sets are grouped into various types. Some of these include finite, infinite, empty/null, single-tone sets, and many more.

Finite set

This is a set with a countable number of elements. For example, set

K = {x:x is an integer between 1 and 10}. That is set K = {2, 3, 4, 5, 6, 7, 8, 9}.

Infinite set

This is a set with an uncountable number of elements. For example, set A= {Counting numbers}.

Null/empty set

This is a set that does not contain any element. An empty set is denoted using the symbol ∅. It is read as 'phi". For example, set B= {children with two biological fathers}. That is set B= { }.

Singleton sets/unit set

This is a set with only one element. For example, set C = {months with less than 30 days}. That is set C= {February}.

Equal sets

If two or more sets have the same members, then they are called equal sets. For example, A= {m, n, r, t}, and B = {n, r, m,t}. Hence, sets A and B are equal sets. This can be represented as A=B.

Unequal sets

If two sets have at least one different member, then they are unequal sets. For example, set

P= {a, b, c, d} and Q= {a, b, c, m} . This can be represented as P≠Q

Equivalent sets

Two sets are equivalent if they have the same number of elements. Here the elements are different. For example, set X = {1, 2, 3, 4} and set Y = {a, b, c, d}. Therefore, sets X and Y are equivalent. This can be represented as n(X) = $n(Y)$.

Disjoint sets

Two sets are disjoint if they do not have any element in common. For example, set

A = {even numbers less than 10}. And set B = {odd numbers less than 10} . That is set

A = {2, 4, 6, 8}. Set B = {1, 3, 5, 7, 9}. Therefore, sets A and B are disjoint.

Overlapping sets

Two sets are said to be overlapping if at least one member belongs to the two sets. For example, set R {Prime numbers less than 10 } and S = {even numbers less than 10} . That is set R = {2, 3, 5, 7} and S = {2, 4, 6, 8}. Here element 2 is in set R as well as set S. therefore, sets R and S are overlapping sets.

Subset and superset

Given two sets P and Q, if every member in set P is present in set Q, then, P is a subset of Q. This is represented by P□Q. On the other hand, Q is a superset of P. This is represented as Q□P. For example, given P = {2, 4, 6, 8, 10} and Q = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} then P□Q since all the members in P are present in set Q. Also, Q□P denotes that set Q is the superset of P.

Power sets

This is the set of all subsets that a set would have. For example, set P = {a, b, c}. The power set of P = {(∅), (a), (b), (c), (a,b), (a,c), (b,c), (a,b,c) .

Therefore, set P has 8 subsets.

Take note

The number of subsets of a given set is = 2^n when n represents the number of members in a set.

Universal set

This is the collection of all the elements concerning a particle subject. This is also known as the mother set. The universal set is denoted by the symbol "u". For example, let V = {1, 2, 3, 4...10} . Here {2, 4, 6, 8, 10}, {1, 3, 5, 7, 9} are subjects of the universal set.

Exercise 3

1. Classify the following sets as finite and infinite sets:
   A= {odd numbers}
   B= {odd numbers less than 20}
   C= {....-2,-1,0,1,2...}
   D= {0,1,2,3,.....}
   E= {1,2,3,.....10}
2. Which of the following are unit sets?
   P= {months with less than 31 days}
   Q = {months with less than 30 days}
   R= {days of the week with names beginning with W}
   S= {O}
3. Which of the following is a null set?
   a. {Months of the year beginning with the letter M}
   b. {Day of the week with names beginning with a letter F}
   c. { }
   d. {0}

Operations on sets

In this section, we shall consider the following operations on sets: intersection, union, and complement of sets.

Intersection of sets

The set of members that belong to sets A and B at the same time is the intersection of A and B. This is denoted as A∩B. The intersection of sets is denoted using the symbol "∩". for example, {2, 4, 6, 8} ∩ {2,3,5} = {2}.

Union of sets

The union of two sets A and B are the list of the members of set A and set B or the members in both sets A and B without repeating the common elements. The members are always listed in ascending order of magnitude. The union of sets A and B is denoted as A∩B. The symbol "∪" denotes the union of sets. For example, Given P = {2,3,5 } and Q= {2, 4, 6, 8}. P ∪ Q = {2, 3, 4, 5, 6,8}.

Complement of sets

The complement of set A which is denoted as A^1 is the set of all elements in the universal set that are not present in set A. Put another way A^1 is denoted as U - A, which is the difference between the elements of the universal set and set A. For example, given U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A= {2, 4, 6, 8, 10}

Find A^1, the complement of A.

Solution

A^1 = U - A

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} – {2, 4, 6, 8, 10}

Therefore, A^1= {1, 3, 5, 7, 9}.

Let us consider the following worked examples:

Example1:

List the members of each of the sets

B= {Whole numbers from 20 to 30} and D = {Factors of 63}.

List the members of

i. B∩D   ii. B∪D

Solution

Listing the members of sets B and D we have

B= {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

D = {1, 3, 7, 9, 21, 63}.

i. B∩D = {21}

ii. B∪D = {1, 3, 7, 9, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 63}.

Example 2:

If X = {Prime numbers less than 13} and Y = {odd numbers less than 13}.

i. List the members of X and Y.

ii. List the member of X∩Y and X∪Y

Solution

i. X = {2, 3, 5, 7, 11}

Y = {1, 3, 5, 7, 9, 11}

ii. X∩Y = {3, 5, 7, 11}

X∪Y = {1, 2, 3, 5, 7, 9, 11}

Example 3:

If P = {7, 11, 13} and Q = {9, 11, 13}. Find P∩Q and P∪Q.

Solution

P∩Q={11, 13}

P∪Q={7, 9, 11, 13}

Example 4:

The set P = {prime numbers less than 20} and Q = {odd numbers less than 10}. Find

(a) P∩Q   (b) P∪Q

Solution

P= {2, 3, 5, 7, 11, 13, 17, 19}

Q= {1, 3, 5, 7, 9}

(a) P∩Q = {3, 5, 7}

(b) P∪Q = {1, 2, 3, 5, 7, 9, 11, 13, 17, 19}

Example 5:

Given the universal set U={a, b, c, d, p, q, r, s}, A= {a, b, c}, A= {a, b, c, p, q, r} are the subsets of U. Find

(a) A∩B   (b) A∪B   (c) A^1 (d) B^1

Solution

a. A∩B = {a, b, c}

b. A∪B = {a, b, c, p, q, r}

c. A^1 = {d, p, q, r, s}

d. B^1 = {d, s}

Exercise 4

1. U= {0, 1}. How many subjects has U?

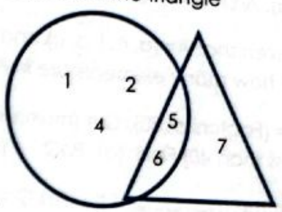
2. If P = {2, 4, 6, 8} and Q = {even counting numbers less than 12}. Find

   a. P∩Q

   b. P∪Q

   c. What is the relationship between P and Q

3. In the diagram below, P is the set of numbers in the circle and Q is the set of numbers in the triangle



What is (a) P∩Q   (b) P∪Q

4. Two sets that have no common members are known as...

5. P = {0, 2, 4, 6} and Q = {1, 2, 4, 5}. Find (a) P∩Q   (b) P∪Q

6. Set A is called ______ of set B when all the members of set A are also members of set B.

7. M= {multiple of 3 between 10 and 20}, N = {Even numbers between 10 and 20}. Find (a) M∩N   (b) M∪V

8. If P= {Multiples of 4 less than 16}. Find P.

9. {1, 3, 5, 7, 9, 11, 13, 15} and R= {1, 2, 3, 5, 6, 7, 10, 11, 12}. Find

   (a) Q∩R   (b) Q∪R

10. If E = {prime numbers between 10 and 20} and

11. F = {odd numbers between o and 16}. Find (a) E∩F   (b) E∪F

12. Given A= {1, 2, 3, 4, 5, 6} and B = {3, 6, 9, 12}. Find (a) A∩B   (b) A∪B

13. If A= {18, 19, 20} and B = {15, 16, 17}. Find: (a) A∩B   (b) A∪B

14. If P = {2, 3, 5, 7} and Q = {2, 4, 6, 8}, find (a) P∩Q   (b) P∪Q

15. If A = {5, 10 15, 20, 25, 125} and B= {5, 10, 15, 20, 25, 625}. List the elements of
    (a). $A \cap B$ (b) $A \cup B$

16. Given that A= {a, e, i, o, u} and B= {r, s, t}, how many elements are in $A \cap B$?

17. P= {Factors of 30}, Q = {multiples of 5 less than 40} Find (a) $P \cap Q$ (b) $P \cup Q$

18. List the members of the set Q = {prime Factors of 30}.

19. Given that P = {m,n,o,p}. How many subsets has set P?

20. If M = {Multiples of 4 between 10 and 25} and N = {even numbers between 11 and 23}. Find (a) $M \cap N$ (b) $M \cup N$

21. If Q = {1, 3, 5, 7, 9, 10, 11, 13,15 } and T= {1, 2, 3, 5, 6, 7, 10, 11, 12}. Find
    (a) $Q \cap T$ (b) $Q \cup T$

22. If A= {2,6,8} and B = {4,6,8,10}. Set A is a subset of B. True /false

23. P= {*x*:*x* is an even number greater than two and less or equal to twelve}. List the members of P.

24. If P = {factors of 36} and Q = {Multiples of 4 less than 40}. Find the number of $P \cap Q$.

25. Given that X= {whole numbers from 4 to 13} and Y = {multiples of 3 between 2 and 20}. Find (a) $X \cap Y$ (b) $X \cup Y$

Properties of sets

In this section, we shall consider the following properties of sets. Commutative, associative, distributive, identity, and complement properties.

Commutative property

Let A and B be two intersecting subsets of the universal set U. we can write the following equations

1. $A \cap B = B \cap A$
2. $A \cup B = B \cup A$

   Therefore, the intersection (∩) and union (∪) of sets are commutative.

   For example, if A = {2, 3, 5, 7} and B= {1, 3, 5} then,

   $A \cap B$ = {3, 5} and $B \cup A$ = {3, 5}

   Therefore, $A \cap B$ = $B \cup A$ = {3, 5}.

   Also, $A \cup B$ = {1, 2, 3, 5, 7} and $B \cup A$ = {1, 2, 3, 5, 7}

   Hence $A \cup B$ = $B \cup A$ = {1, 2, 3, 5, 7}.

Associative property

Let A, B, and C be three intersecting sets, then

1. $(A \cap B) \cap C = A \cap (B \cap C)$
2. $(A \cup B) = A \cup (B \cup C)$

Therefore, the intersection (∩) and union (∪) of sets are associative.

For example. Given

A= {2, 4, 6, 8, 10}

B= {1, 2, 3, 4}

C= {2, 3, 5, 7}

$(A \cap B) \cap C = A \cap (B \cap C)$

From the left-hand side,

$(A \cap B) \cap C$

Bracket first, $A \cap B$ = {2, 4}

$(A \cap B) \cap C$ = {2,4} ∩ {2, 3, 5, 7}

= {2}

Therefore, $(A \cap B) \cap C$ = {2}

Also from the R. H. S

$A \cap (B \cap C)$

Bracket first,

$(B \cap C)$ = {2, 3}

$A \cap (B \cap C)$ = {2, 4, 6, 8, 10} ∩ {2, 3}

Therefore, $A \cap (B \cap C)$ = {2}

Hence $(A \cap B) \cap C = A \cap (B \cap C)$ = {2}

Also, $(A \cap B) \cup C = A \cup (B \cup C)$

From the L.H.S.

Bracket first

$A \cup B$ = {1, 2, 3, 4, 6, 8, 10}

$(A \cup B) \cup C$ = {1, 2, 3, 4, 6, 8, 10} ∪ {2, 3, 5, 7}

= {1, 2, 3, 4, 5, 6, 7, 8, 10}

Therefore, $(A \cup B) \cup C$ = {1,2,3,4,5,6,7,8,10}

From the R. H. S

$A \cup (B \cup C)$

Bracket first

$(B \cup C)$ = {1, 2, 3, 4, 5, 7}

$A \cup (B \cup C)$ = {2, 4, 6, 8, 10} ∪ {1, 2, 3, 4, 5, 7}

= {1, 2, 3, 4, 5, 6, 7, 8, 10}

Therefore, $A \cup (B \cup C)$ = {1, 2, 3, 4, 5, 6, 7, 8,10}

Hence, $(A \cup B) \cup C = A \cup (B \cup C)$ = {1, 2, 3, 4, 5, 6, 7, 8, 10}

Distributive property

Let A, B, and C be three intersecting subjects. Then,

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   Therefore, intersection (∩) is distributive over the union (∪) of sets.

2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   Therefore, the union (∪) of the sets is distributive over the intersection (∩) of sets For example,

   Given

   A= {2, 4, 6}, B = {2, 3, 6}, and C = {3, 6, 9}, then, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

From the LHS

$A \cap (B \cup C)$

Bracket first.

$(B \cup C)$ = {2, 3, 6, 9}

$A \cap (B \cup C)$ = {2, 4, 6} ∩ {2, 3, 6, 9}

Therefore, $A \cap (B \cup C)$ = {2, 6}

From the R.H.S

$(A \cap B) \cup (A \cap C)$

$(A \cap B)$ = {2, 6}

$(A \cap C)$ = { 6}

$(A \cap B) \cup (A \cap C)$ = {2, 6} ∪ { 6}

= {2, 6}

Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ = {2, 6}

Activity 1

Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Identity property

$A \cup \emptyset = A$

$A \cap U = A$

Where $\emptyset$ is a null set, and U is a universal set.

Complement property

$A \cup A^1 = U$

Where A^1 is the complement of A. U is the universal set.

Activity 2

Given that

P = {2, 4, 6, 8, 10}

Q= {4, 8, 12, 16}

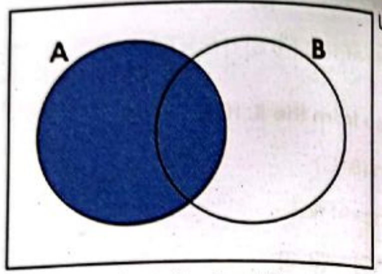
R= {1, 2, 3, 4, 5, 8}

Show that

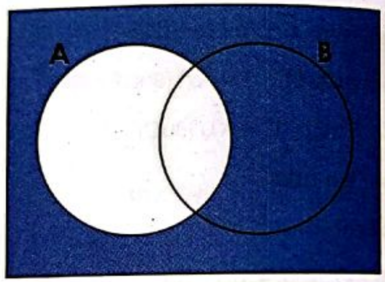
1. $A \cap B = B \cap A$
2. $A \cup B = B \cup A$
3. $(A \cap B) \cap C = A \cap (B \cap C)$
4. $(A \cup B) \cup C = A \cup (B \cup C)$
5. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Operations of Sets and Venn Diagram

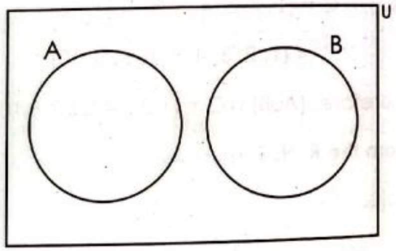
A Venn diagram is also referred to as a set diagram or a logical diagram that shows different set operations such as the union of sets, the intersection of sets, and the differences between sets. It can also be used to show subsets of a set. For example, the set of whole numbers is a subset of integers. Let us describe the various regions of a Venn diagram.



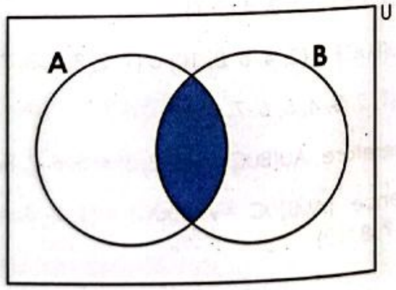
The shaded portion is set A



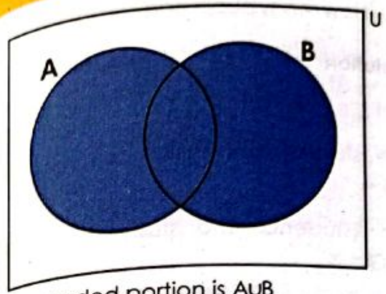
The shaded portion is A^1



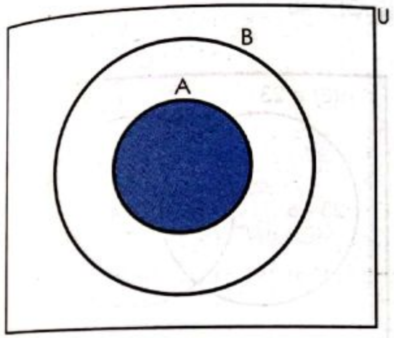
A and B are disjoint sets



The shaded portion is $A \cap B$



The shaded portion is $A \cup B$



A□B

Two Set Problems

Let us consider the Venn diagram below

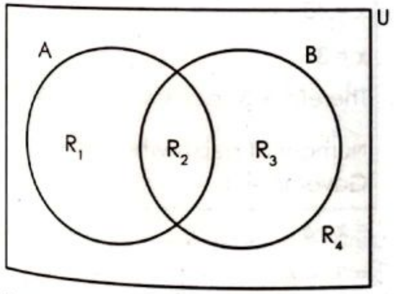


Figure 1.

For any two intersecting sets A and B,

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
3. $n(A) = n(A \cup B) + n(A \cap B) - n(B)$
4. $n(B) = n(A \cup B) + n(A \cap B) - n(A)$

From figure 1 above,

Number of universal set $n(U) = R_1+R_2+R_3+R_4$

Note:

If there is no complement, then R4 = 0. Therefore, $n(U) = R_1 + R_2+R_3$

Let us consider the following worked examples:

Example 1:

In a class of 60 students 46 passed mathematics and 42 passed the English language. Every student passed at least one of the two subjects.

i. Illustrate the information on a Venn diagram.

ii. How many students passed both subjects?

iii. How many passed exactly one subject?

Let n represent the number of students who passed both subjects.

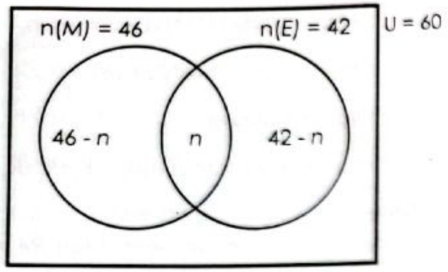
Solution

Let U = {students in the class}, n(U) = 60

M = {students who passed mathematics }, n(M)= 46

E= {students who passed the English},n(E)= 42

N = the number of students who passed both subjects



i. The sum of the three regions = U

$46 - n + n + 42 - n = 60$

$88 - n = 60$

$n = 88 - 60$

$n = 28$

Therefore, 28 students passed both subjects

ii. Students who passed exactly one

subject $= 46 - n + 42 - n$

But $n = 28$

$= 46 - 28 + 42 - 28$

$= 18 + 14$

$= 32$

Example 2:

In a secondary school class, 23 pupils study Economics, 6 pupils' study both Government and Economics. 48 pupils' study either Government or Economics or both.

a. Represent the above information on a Venn diagram.

b. What is the total number of pupils who study Government?

c. How many pupils study only Government?

d. How many pupils study only Economics?

Solution

Let U = {secondary school students}

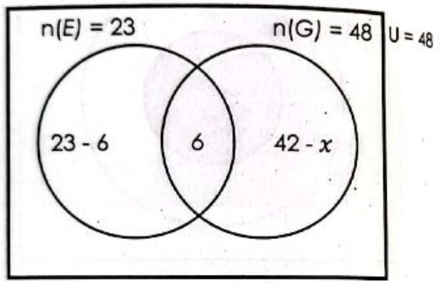
E = {students who study Economics}, n(E)= 23

G= {students who study Government}, n(G)= x

$n(E\cap G) = 6$

$n(E\cup G) = 48$

a.



b. Sum the three regions:

$23 - 6 + 6 + x - 6 = 48$

$17 + x = 48$

$x = 48 - 17$

$x = 31$

Therefore, 31 pupils study Government.

c. Number of pupils who study only Government

$= x - 6$

$= 31 - 6$

$=25$

d. Only Economic $= 23 - 6$

$= 17$

Therefore, 17 pupils study only Economics.

Example 3:

There are 20 students in a hostel, 16 of them are fluent in French, and 10 of them are fluent in the English language. Each student is fluent in at least one of the two languages.

i. Illustrate this information on a Venn diagram.

ii. How many students are fluent in both English and French?

iii. How many students are fluent in exactly one language?

Solution

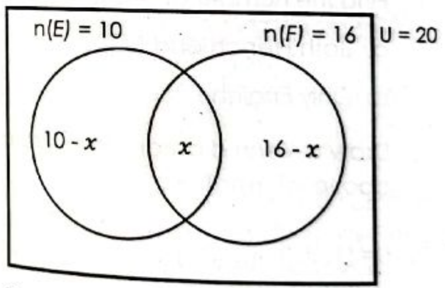
Let x = number of students who are fluent in both English and French

U= {students in hostel}, n(U) = 20

E = {the students who are fluent in English}, n(E) = 10

F= {students who are fluent in French }, n(F) = 16

i. The Venn diagram is shown below



ii. The sum of members in the three regions = U

$20 = (10 - x) + x + (16 - x)$

$20 = 10 - x + x + 16 - x$

$20 = 26 - x$

$x = 26 - 20 \Rightarrow x = 6$

Therefore, the number of students who are fluent in both languages is 6

iii. Exactly one language

$= (10 - x) + (16 - x)$

But $x = 6$

$= (10 - 6) + (16 - 6)$

$= 4 + 10 = 14$

Therefore, 14 students are fluent in both languages

Example 4:

Twenty-five (25) students in a class took an examination in Mathematics and Science. 17 of them passed in science and 8 passed in both Mathematics and Science. 3 students did not pass any of the subjects.

i. How many passed in Mathematics?

ii. How many passed in only one subject?

Solution

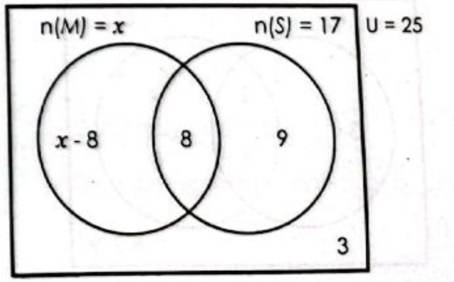
Let U = {students in a class},n(U) = 25

M = {students who passed mathematics}, n(M) = x

S= {Students who passed science}, n(S) = 17

M∩S= {students who passed both subjects}, $n(M\cap S)= 8$

$n(M\cup S)' = 3$



i. How many passed in Mathematics?

$x - 8 + 8 + 9 + 3 = 25$

$x + 12 = 25$

$x = 25 - 12$

$x = 13$

Therefore, 13 pass Mathematics

ii. Students who passed only one subject

$= (x - 8) + 9$

$= (13 - 8) + 9$

$= 5 + 9 = 14$

Therefore 14 students passed only one subject

Example 5:

M is a set consisting of all positive integers between 1 and 10. P and Q are subsets of M such that P = {factors of 6} and Q = {multiple of 2}.

i. List the elements of M, P, and Q.

ii. Represent M, P, and Q on a Venn diagram.

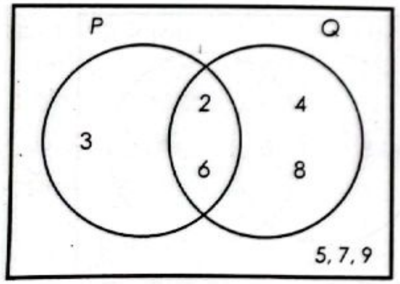
iii. Find $P \cap Q$.

Solution

i. M = {2, 3, 4, 5, 6, 7, 8, 9}

P = {2, 3, 6}

Q = {2, 4, 6, 8}



(iii) $P \cap Q$ = {2, 6}.

Exercise 5

Solve the following questions:

1. In a class of 40 students, 11 speak Twi only, 7 speak Ga only, and 5 speak neither Twi nor Ga.
   a. Represent the above information on a Venn diagram.
   b. How many students speak Twi and Ga?
   c. How many speak Twi?
   d. How many students speak Ga?
   e. How many students speak exactly one language?

2. There are 50 pupils in a class. Out of this number $\frac{1}{10}$ speak French only and $\frac{4}{5}$ of the remainder speak both French and English. If the rest speak English only.
   i. Find the number of students who speak
      a. Both French and English.
      b. Only English.
   ii. Draw a Venn diagram to illustrate the above information.

3. U= {1, 2, 3, 4.....18};
   A= {prime numbers} and
   B= {odd numbers greater than 3}
   a. If A and B are subjects of the universal set, U, list the members of A and B.
   b. Find the set
      i. $A \cap B$ ii. $A \cup B$
   c. Illustrate U, A, and B on a Venn diagram.

4. There are 30 boys in a sporting club. 20 of them play hockey and 15 play volleyball. Each boy lays at least one of the two games.
   i. Illustrate the information on a Venn diagram.
   ii. How many boys play volleyball only?

5. E and F are subjects of the universal set U such that
   N = {natural numbers less than 15}
   E = {even numbers between 1 and 15}
   F = {multiples of 4 between 9 and 15}
   i. List the elements of U, E, and F.
   ii. Draw a Venn diagram to show the sets U, E, and F.

6. Sets A and B are subjects of a universal set U = {1, 2, 3, 4, 5,......18} such that A = {Even numbers} and B = {multiples of 3}.
   i. List the elements of sets A, B, $(A \cap B)$, $(A \cup B)$, and $(A \cup B)^1$.
   ii. Illustrate the information in (i) on a Venn diagram.

7. In a school of 255 students, 80 of them study Arabic only, and 125 study French only. Each student studies at least one of the two subjects.
   i. Draw a Venn diagram to represent the information.
   ii. How many students study
      a. Both subjects?
      b. French?

8. Eighty farmers in a village grow either cassava or yam. Fifty of them grow yam while 60 grow cassava. Each farmer grows at least one of the two crops.
   a. Represent the above information on a Venn diagram.
   b. Use the Venn diagram to find farmers who grow
      i. Both crops.
      ii. Only one crop.

9. Fifty students in a class took an examination in French and mathematics. If 14 of them passed French only, 23 passed in both French and mathematics, and 5 of them failed in both subjects, find
   i. The number of students who passed in French.
   ii. The number of students who passed in only one subject.

10. In a class of students who offer Mathematics or English. Eighteen of them offer only Mathematics, 3 offer English and 7 offer both Mathematics and English.
    a. Represent the above information on a Venn diagram.
    b. Use the Venn diagram to find the number of students who
       i. Offer Mathematics.
       ii. Offer only one subject.
       iii. Offer either Mathematics or English.

i. How many passed in Mathematics?

$x - 8 + 8 + 9 + 3 = 25$

$x + 12 = 25$

$x = 25 - 12$

$x = 13$

Therefore, 13 pass Mathematics

ii. Students who passed only one subject

$= (x - 8) + 9$

$= (13 - 8) + 9$

$= 5 + 9 = 14$

Therefore 14 students passed only one subject

Example 5:

M is a set consisting of all positive integers between 1 and 10. P and Q are subsets of M such that P = {factors of 6} and Q = {multiple of 2}.

i. List the elements of M, P, and Q.

ii. Represent M, P, and Q on a Venn diagram.

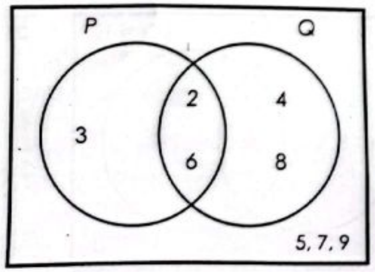
iii. Find $P \cap Q$.

Solution

i. M = {2, 3, 4, 5, 6, 7, 8, 9}

P = {2, 3, 6}

Q = {2, 4, 6, 8}



(iii) $P \cap Q$ = {2, 6}.

Exercise 5

Solve the following questions:

1. In a class of 40 students, 11 speak Twi only, 7 speak Ga only, and 5 speak neither Twi nor Ga.
   a. Represent the above information on a Venn diagram.
   b. How many students speak Twi and Ga?
   c. How many speak Twi?
   d. How many students speak Ga?
   e. How many students speak exactly one language?

2. There are 50 pupils in a class. Out of this number $\frac{1}{10}$ speak French only and $\frac{4}{5}$ of the remainder speak both French and English. If the rest speak English only.
   i. Find the number of students who speak
      a. Both French and English.
      b. Only English.
   ii. Draw a Venn diagram to illustrate the above information.

3. U= {1, 2, 3, 4.....18};
   A= {prime numbers} and
   B= {odd numbers greater than 3}
   a. If A and B are subjects of the universal set, U, list the members of A and B.
   b. Find the set
      i. $A \cap B$ ii. $A \cup B$
   c. Illustrate U, A, and B on a Venn diagram.

4. There are 30 boys in a sporting club. 20 of them play hockey and 15 play volleyball. Each boy lays at least one of the two games.
   i. Illustrate the information on a Venn diagram.
   ii. How many boys play volleyball only?

5. E and F are subjects of the universal set U such that
   N = {natural numbers less than 15}
   E = {even numbers between 1 and 15}
   F = {multiples of 4 between 9 and 15}
   i. List the elements of U, E, and F.
   ii. Draw a Venn diagram to show the sets U, E, and F.

6. Sets A and B are subjects of a universal set U = {1, 2, 3, 4, 5,......18} such that A = {Even numbers} and B = {multiples of 3}.
   i. List the elements of sets A, B, $(A \cap B)$, $(A \cup B)$, and $(A \cup B)^1$.
   ii. Illustrate the information in (i) on a Venn diagram.

7. In a school of 255 students, 80 of them study Arabic only, and 125 study French only. Each student studies at least one of the two subjects.
   i. Draw a Venn diagram to represent the information.
   ii. How many students study
      a. Both subjects?
      b. French?

8. Eighty farmers in a village grow either cassava or yam. Fifty of them grow yam while 60 grow cassava. Each farmer grows at least one of the two crops.
   a. Represent the above information on a Venn diagram.
   b. Use the Venn diagram to find farmers who grow
      i. Both crops.
      ii. Only one crop.

9. Fifty students in a class took an examination in French and mathematics. If 14 of them passed French only, 23 passed in both French and mathematics, and 5 of them failed in both subjects, find
   i. The number of students who passed in French.
   ii. The number of students who passed in only one subject.

10. In a class of students who offer Mathematics or English. Eighteen of them offer only Mathematics, 3 offer English and 7 offer both Mathematics and English.
    a. Represent the above information on a Venn diagram.
    b. Use the Venn diagram to find the number of students who
       i. Offer Mathematics.
       ii. Offer only one subject.
       iii. Offer either Mathematics or English.

11. In a class of 70 students, 40 belong to the Red Cross Society, 27 belong to the Girls' Guide Society and 12 belong to both clubs. The remaining students do not belong to any of the two societies.

    a. Illustrate the information on a Venn diagram.

    b. How many students belong to the Red Cross Society only?

    c. How many students do not belong to any of the two societies?

12. a) In an examination, 50 candidates sat for either Mathematics or English language. 60% passed in Mathematics 48% passed in the English language. If each candidate passed in at least one of the subjects, how many candidates passed in?

    i. Mathematics

    ii. English language

    b) Illustrate the information given in (a) on a Venn diagram

    c) Using the information on a Venn diagram, find the number of candidates who passed

    i. Both subjects.

    ii. Mathematics only.

13. In a class of 30 girls, 17 play football, 12 play hockey, and 4 play both games.

    i. Draw a Venn diagram to illustrate the given information.

    ii. How many girls play

    a. One or two of the games?

    b. None of the two games?

14. In an examination, 60 candidates passed Integrated Science or Mathematics. If 15 passed both subjects and 9 more passed mathematics than Integrated Science, find the

    (i) The number of candidates who passed in each subject.

    (ii) The number of candidates that passed exactly one subject.

STRAND 1: NUMBER

SUB-STRAND 2: NUMBER OPERATIONS

LESSON 1:

Multiplication facts up to 144

In this section, we shall deal with a multiplication facts table with individual numbers from 1 to 12. Let us consider the multiplication chat below

Table 1: Multiplication facts up to 144

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

From Table 1 above, each row in the multiplication chart gives a times table for a given number. The rows give the result when a given number is multiplied by other numbers.

Example1: Row " 8"

It indicates that when 8 is multiplied by 1 you get 8, by 2 give 16, by 3 give 24, by 4 give 32, by 5 give 40, etc. Row 8 can be used to generate the 8 times table as shown below

8 x 1 = 8
8 x 2 = 16
8 x 3 = 24
8 x 4 = 32
8 x 5 = 40
8 x 6 = 48
8 x 7 = 56
8 x 8 = 64
8 x 9 = 72
8 x 10 = 80
8 x 11 = 88
8 x 12 = 96

Example 2: Row "10"

Row 10 shows that when 10 is multiplied by 1 you get 10, by 2 you get 20, by 3 you get 30, by 4 you get 40, by 5 you get 50, etc. Let us consider row 10 to generate a 10-times table as illustrated below.

10 x1 = 10
10 x 2 = 20
10 x 3 = 30
10 x 4 = 40
10 x 5 = 50
10 x 6 = 60
10 x 7 = 70
10 x 8 = 80
10 x 9 = 90
10 x 10 100
10x 11 = 110
10x 12 = 120

Exercise 1

Use the multiplication chart in Table 1 to generate the multiplication times table for each of the following numbers:

a. 4 times table
b. 5 times table
c. 6 times table
d. 7 times table
e. 9times table
f. 11 times table
g. 12 times table

Exercise 2

Complete the following:

a)	2 x 3	=		k)	6 x 8	=	
b)	2 x 6	=		l)	6 x 12	=	
c)	2 x 8	=		m)	7 x 8	=	
d)	2 x12	=		n)	7 x 9	=	
e)	3 x 4	=		p)	7 x 11	=	
f)	3 x 7	=		o)	8 x 4	=	
g)	3 x 9	=		p)	8 x 9	=	
h)	4 x 6	=		q)	8 x 12	=	
i)	4 x 12	=		r)	9 x 12	=	
j)	6 x 7	=		s)	12 x12	=	

Division as inverse of multiplication

Every division sentence has a related multiplication sentence in it. In this lesson, we shall consider the relationship between division and multiplication. Let us consider the following examples:

Example 1: Find the value of 20 ÷ 2

Solution

20 ÷ 2 = ☐

This means that 2 x ☐ = 20

Since 2 x 10 = 20

Then, 20 ÷ 2 = 10

Example 2: Solve 729

Solution

72 ÷ 9 = ☐

This means that 9 x ☐ = 72

Since 9 x 8 = 72

Then 72 ÷ 9 = 8

Example 3: evaluate 132

Solution

132 ÷ 12

This means that 132 ÷ 12 = ☐

Since 12 x 11 = 132

Then 132 ÷ 12 = 11

Example 4: Calculate 84

Solution

84 ÷ 12 = ☐

This means that 12 x ☐ = 84

Since 12 x 7 = 84

Then 84 ÷ 12 = 7

Example 5: Calculate the value of 108

Solution

108 ÷ 9 = ☐

This means that 9 x ☐ = 108

Since 9 x 12 = 108

Then, 108 ÷ 9 = 12

Exercise 3

Complete the following statements:

a. 20 ÷ 4 = ☐	⟶	4 x ☐ =	20
b. 28 ÷ 7 = ☐	⟶	7 x ☐ =	28
c. 56 ÷ 8 = ☐	⟶	8 x ☐ =	56
d. 63 ÷ 7 = ☐	⟶	7 x ☐ =	63
e. 45 ÷ 9 = ☐	⟶	9 x ☐ =	45
f. 42 ÷ 6 = ☐	⟶	6 x ☐ =	42
g. 55 ÷ 5 = ☐	⟶	5 x ☐ =	55
h. 96 ÷ 12 = ☐	⟶	12 x ☐ =	96
i. 48 ÷ 4 = ☐	⟶	4 x ☐ =	48
j. 110 ÷ 10 = ☐	⟶	10 x ☐ =	110

Exercise 4

Convert each of the following multiplication sentences into two division sentences:

For Example

7 × 8 = 56 56 ÷ 8 = 7
56 ÷ 7 = 8

a. 9 x 4 = 36 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

b. 12 × 4 = 48 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

c. 8 × 5 = 40 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

d. 6 × 8 = 48 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

e. 7 × 5 = 35 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

f. 9 × 6 = 54 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

g. 11 × 6 = 66 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

h. 12 × 5 = 60 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

i. 12 x 7 = 84 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

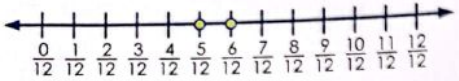
j. 8 x 11 = 88 ☐ ÷ ☐ = ☐
☐ ÷ ☐ = ☐

Decimal names of given bench mark fractions

Bench mark fractions are common fractions that we can measure or evaluate when measuring, comparing, or ordering other fractions. Benchmark fractions are easy to visualize and identify. Hence, they help in estimating parts. Put another way, benchmark fractions are commonly used fraction that is used for comparing other fractions. Examples of benchmark fractions are:

$\frac{1}{2}$, 0, 1, $\frac{1}{3}$, $\frac{1}{10}$ etc

For example, we can compare $\frac{5}{12}$ and -
using benchmarks and a number line.



From the number line, $\frac{5}{12} < \frac{6}{12}$

Since $\frac{6}{12} = \frac{1}{2}$

Therefore, $\frac{5}{12} < \frac{6}{12}$

The decimal names of some benchmark fractions as well as a percentage are summarized in the following table:

Fractions	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Decimal	0.25	0.5	0.75	1.0
Percentage	25%	50%	75%	100%

Converting Common Fractions to Decimals

To convert fractions into decimals, divide the numerator by the denominator. Let us consider the following examples:

Example 1:

convert the following common fractions into decimals:

a). $\frac{3}{4}$ b) $\frac{3}{5}$ c) $\frac{1}{2}$ d) $\frac{3}{10}$ e) $\frac{3}{5}$

Solution

$$\text{a. } \frac{3}{4} = 4\,\square\begin{array}{r} 0.75 \\ \hline 30 \\ -28 \\ \hline 20 \\ -20 \\ \hline 00 \end{array}$$

Therefore, $\frac{3}{4} = 0.75$

$$\text{b. } \frac{3}{5} = 5\,\square\begin{array}{r} 0.6 \\ \hline 30 \\ -30 \\ \hline 00 \end{array}$$

Therefore, $\frac{3}{5} = 0.6$

$$\text{c. } \frac{1}{2} = 2\,\square\begin{array}{r} 0.5 \\ \hline 10 \\ -10 \\ \hline 00 \end{array}$$

$$\text{d. } \frac{3}{10} = 10\,\square\begin{array}{r} 0.3 \\ \hline 30 \\ -30 \\ \hline 00 \end{array}$$

Therefore, $\frac{3}{10} = 0.3$

$$\text{e. } \frac{2}{5} = 5\,\square\begin{array}{r} 0.4 \\ \hline 20 \\ -20 \\ \hline 00 \end{array}$$

Therefore, $\frac{2}{5} = 0.4$

Exercise 5

Convert the following fractions into decimals:

a. $\frac{1}{3}$ f) $\frac{4}{5}$

b. $\frac{2}{3}$ g) $\frac{3}{8}$

c. $\frac{1}{5}$ h) $\frac{5}{8}$

d. $\frac{2}{5}$ i) $\frac{7}{8}$

e. $\frac{3}{5}$ j) $\frac{5}{9}$

Converting Decimals to Fractions

To convert decimals into fractions, create a fraction with the decimal as the numerator and 1 as the denominator. Then multiply both the numerator and denominator by ten many times to get whole numbers for the numerators and denominator. Then simplify both terms.

Example 1.

Convert the following decimals to fractions

(a) 0.75 (b) 0.625 (c) 0.4

(d) 0.25 (e) 0.8

Solution

a) $0.75 = \frac{0.75}{1}$

$= \frac{75}{100}$

$= \frac{3}{4}$

Therefore, $0.75 = \frac{3}{4}$

b. $0.625 = \frac{0.625}{1}$

$= \frac{625}{1000}$

$= \frac{25}{40}$

$= \frac{5}{8}$

c. $0.4 = \frac{0.4}{1}$

$= \frac{4}{10}$

$= \frac{2}{5}$

d. $0.25 = \frac{0.25}{1}$

$= \frac{25}{100}$

$= \frac{1}{4}$

e. $0.8 = \frac{0.8}{1}$

$= \frac{8}{10}$

$= \frac{4}{5}$

Exercise 6

Convert the following decimals to fractions

a) 0.5 f) 0.625

b) 0.2 g) 0.875

c. 0.6 h) 0.125

d. 0.16 i) 0.375

e. 0.1 j) 0.0625

Converting Common Fractions to Percentages

Consider the following steps in converting common fractions to percentages:

i. Use the division to convert the fraction to a decimal. For example, $\frac{1}{2} = 1 \div 2 = 0.5$

ii. Multiply by 100 to get the percent value. To multiply by 100, move the decimal points 2 steps to the right.

Therefore, 0.5 x 100 = 50%.

Worked examples

Convert the following common fractions to percentages:

a) $\frac{3}{4}$ b) $\frac{7}{8}$ c) $\frac{4}{5}$ d) $\frac{2}{5}$ e) $\frac{3}{8}$

Solution

$$\text{a. } \frac{3}{4} = 4\,\square\begin{array}{r} 0.75 \\ \hline 30 \\ -28 \\ 20 \\ 20 \\ 00 \end{array}$$

Therefore $= \frac{3}{4} 0.75$

$\frac{3}{4} \times 100\% = 0.75 \times 100\%$

Therefore, $\frac{3}{4} = 75\%$

$$\text{b. } \frac{7}{8} = 8\,\square\begin{array}{r} 0.875 \\ \hline 70 \\ -64 \\ 60 \\ -56 \\ 40 \\ -40 \\ 00 \end{array}$$

Therefore, $\frac{7}{8} = 0.875$

$\frac{7}{8} \times 100\% = 0.875 \times 100\%$

$= 87.5\%$

c. $\frac{4}{5} = 5\square\begin{array}{|l} \overline{0.8} \\ 40 \\ -40 \\ 00 \end{array}$

Therefore $\frac{4}{5} \times 100\% = 0.8 \times 100\%$

$= 80\%$

d. $\frac{2}{5} = 5\square\begin{array}{|l} \overline{0.4} \\ 20 \\ -20 \\ 00 \end{array}$

Therefore, $\frac{2}{5} \times 100\% = 0.4 \times 100\% = 40\%$

e. $\frac{3}{8} = 8\square\begin{array}{|l} \overline{0.375} \\ 30 \\ -24 \\ 60 \\ -56 \\ 40 \\ -40 \\ 00 \end{array}$

Therefore, $\frac{3}{8} = 0.375$

$\frac{3}{8} \times 100\% = 0.375 \times 100\%$

$= 37.5\%$

Exercise 7

Convert the following common fractions to percentages:

a) $\frac{8}{5}$ f) $\frac{4}{9}$ k) $\frac{11}{3}$

b) $\frac{5}{8}$ g) $\frac{2}{10}$ l) $\frac{8}{15}$

c) $\frac{1}{2}$ h) $\frac{8}{3}$ m) $\frac{9}{4}$

d) $\frac{1}{4}$ i) $\frac{7}{9}$ n) $\frac{4}{7}$

e) $\frac{1}{5}$ j) $\frac{6}{7}$ o) $\frac{7}{11}$

Converting Percentages to Common Fractions

To convert a given percentage to a common fraction, write down the number and divide by 100. Let us consider the following examples:

Worked examples

a) 38% b) 30% c) 48%

d) 0.15% e) 40%

Solution

a. $38\% = \frac{38}{100}$

$= \frac{19}{15}$

b. $30\% = \frac{30}{100}$

$= \frac{3}{10}$

c. $48\% = \frac{48}{100}$

$= \frac{12}{25}$

d. $0.15\% = \frac{0.15}{100}$

$= \frac{15}{10000}$

$= \frac{3}{2000}$

e. $40\% = \frac{40}{100}$

$= \frac{2}{5}$

Exercise 8

Convert the following percentages to common fractions:

a. 56% f) 86% k) 36.2%

b. 24% g) 35% l) 50%

c. 6% h) 60% m) 28%

d. 25% i) 32% n) 10%

e. 80% j) 45% o) 20%

Exercise 9

1. Express 0.625 as a fraction in its lowest term.
2. Express $\frac{3}{8}$ as a percentage.
3. Find $2\frac{1}{2}\%$ of GH₵2,000.00
4. Express 25 as a percentage of 75.
5. Express $\frac{5}{8}$ as a decimal fraction.
6. Express $\frac{1}{25}$ as a decimal fraction.
7. Express $\frac{2}{5}$ as a percentage.
8. In an examination 154 out of 175 candidates passed. What percentage failed?
9. Express 0.68 as a fraction in its lowest term.
10. Express 0.55 as a fraction in its lowest term.
11. Find $12\frac{1}{2}\%$ of GH₵80.00
12. Express 5 as a percentage of 4.
13. Express 0.125 as a fraction in its lowest form.
14. Express 3.75 as a mixed fraction.
15. Express 1.25 as a percentage.
16. Express $\frac{10}{32}$ as a decimal fraction.
17. Express 30% as a fraction in its lowest term.
18. Express $\frac{12}{25}$ in decimal fractions.
19. Express 75% as a fraction in its lowest term.
20. Express 12.5% as a fraction in its lowest term.

Multiplying decimals by 10, 100, 1000, $\frac{1}{25}$ $\frac{1}{100}$, etc

In this section, we shall consider multiplying decimals by 10, 100, 1000, $\frac{1}{10}$, $\frac{1}{100}$, etc.

Let us consider the following steps when multiplying decimals by 10 and its powers

Steps

1. To multiply a decimal by 10, move the decimal point in the multiplicand by one place to the right.
2. To multiply a decimal by 100, move the decimal point in the multiplicand by two places to the right.
3. To multiply a decimal by $\frac{1}{10}$, move the decimal point in the multiplicand by one place to the left.
4. To multiply a decimal by $\frac{1}{100}$, move the decimal point in the multiplicand by two places to the left, etc.

Let us consider the following worked examples:

Examples 1:

Evaluate 734.5 x 10

Solution

734.5 x 10 = 7345

Here we multiplied the number 734.5 by 10, so we move one place to the right

Example 2.

Solve the following:

a. 83.5 100

b. 100.8 1000

c. 89.62 100

d. 0.57 1000

Solution

a. 83.5 100 = 8350

Here we multiplied the number 83.5 by 100 so we move 2 places to the right

Or

$$83.5 \times 100 = \frac{8350}{100} \times 100$$
$$= \frac{8350}{1}$$
$$= 8350$$

b. 100.8 x 1000 = 100800

Here we multiplied 100.8 by 1000, so we move 3 places to the right

Or

$$100.8 \times 1000 = \frac{100800}{1000} \times 1000$$
$$= \frac{100800}{1}$$
$$= 100800$$

c. 89.62 100 = 8962

d. 0.57 1000 = 570

Example 3:

calculate the following:

a. $835.6 \times \frac{1}{10}$

b. $75.468 \times \frac{1}{100}$

c. $0.49 \times \frac{1}{1000}$

d. $26.89 \frac{1}{100}$

Solution

a. $835.6 \times \frac{1}{10} = 83.56$

Here we multiplied the number 835.6x $\frac{1}{10}$, so we move one place to the left

b. $75.468 \times \frac{1}{100} = 0.75468$

Here we multiplied the number 75.468 by $\frac{1}{100}$, so we move two places to the left.

c. $0.49 \times \frac{1}{1000} = 0.00049$

Here we multiplied the number 0.49 by $\frac{1}{1000}$, so we move 3 places to the left.

d. $26.89 \times \frac{1}{100} = 0.2689$

Exercise 10

Solve the following:

a. 384.6 x 10

b. 29.86 x 100

c. 379. 14 x 1000

d. 8514.437 x 10

e. 42.7 x 10

f) 878.4 x 1000

g) 5.879 x 10

h) 5.61 x 100

i) 14.469 x 100

j) 841.35 x 100

Exercise 11

Evaluate the following

a. $48.67 \times \frac{1}{10}$

b. $1.49 \times \frac{1}{100}$

c. $2.635 \times \frac{1}{1000}$

d. $0.7349 \times \frac{1}{100}$

e. $324.49 \times \frac{1}{1000}$

f) $62.7 \times \frac{1}{100}$

g) $72.849 \times \frac{1}{1000}$

i) $5034.52 \times \frac{1}{1000}$

j) $0.49 \times \frac{1}{10}$

k) $41678.4 \times \frac{1}{1000}$

LESSON 2

Commutative properties of addition and multiplication

In this section, we shall consider the commutative properties of addition and multiplication. For any two real numbers p and q:

i. $p + q = q + p$

For example, 20 + 30 = 30 + 20 = 50

ii. $p \times q = q \times p$

For example; 6 5 = 5 6 = 30

Example 1:

Given the two numbers 10 and 5, find

a. 10 + 5

b. 5 + 10

c. What is the relationship between (a) and (b) above?

d. What property is used in (a) and (b) above

Solution

a. 10 + 5 = 15

b. 5 +10 = 15

c. 10 + 5 = 5 + 10

d. The result in (c) shows the commutative property of addition.

Example 2:

Given the two numbers: 7 and 9. Evaluate

a. 7 x 9

b. 9 x 7

c. What is the relationship between the results in (a) and (b) above?

d. What property is shown in (c) above?

Solution

a) 7 x 9 = 63

b) 9 x 7 = 63

c) 7 x 9 = 9 x 7

d) Results in (c) show the commutative property of multiplication.

Associative properties of addition and multiplication

In this section, we shall consider the associative properties of addition and multiplication. For any three numbers a, b, and c;

i. $(a + b) + c = a + (b + c)$

For example

(2+3) +4 =2+(3 +4) = 9

ii. $(a \times b) \times c = a \times (b \times c)$

For example

(2 × 3) × 4 = 2 × (3 × 4) = 24

Let us consider the following worked examples

Example 1:

Given the three numbers 4, 5, and 6. Evaluate

a. (4 + 5) + 6

b. 4 + (5 + 6)

c. What is the relationship between the results in (a) and (b) above?

d. What property is shown (c) above?

Solution

a. (4 + 5) + 6 = 15

b. 4 + (5 + 6) = 15

c. (4 + 5) + 6 = 4 + (5 +6)

d. The associative property of addition is shown in (c)

Example 2:

Given the three numbers 5, 6, and 7. Evaluate

a. (5 x 6) x 7