STRAND 4

DATA

SUB-STRAND 1: DATA AND PROBABILITY

Content Standards

B7.4.1.1 Select, justify, and use appropriate methods to collect data (quantitative and qualitative), display and analyze the data (grouped/ungrouped) presented in frequency tables, line graphs, pie graphs, bar graphs or pictographs and use these to solve and/or pose problems.
B7.4.1.2 Determine the measures of central tendency (mean, median, mode) for a given ungrouped data and use it to solve problems.

SUB-STRAND 2: MEASUREMENT

Content Standards

B7.4.2.1 Identify the sample space for a probability experiment involving single events and express the probabilities of given events as fractions, decimals, percentages and/or ratios to solve problems

26 DATA COLLECTION

STRAND 4: DATA

SUB-STRAND 1: Data and Probability

CONTENT STANDARD: B7.4.1.1 Select, justify, and use appropriate methods to collect data (quantitative and qualitative), display and analyze the data (grouped/ ungrouped) presented in frequency tables, line graphs, pie graphs, bar graphs or pictographs and use these to solve and/or pose problems.

INDICATOR B7.4.1.1.1- Select and justify a method to collect data (quantitative and qualitative) to answer a given question.

B7.4.1.1.2- Design and administer a questionnaire for collecting data to answer a given question(s) and record the results.

Data is a collection of facts such as numbers, values and observations relating to a particular subject.

It enables users to draw meaningful conclusions and make appropriate decisions.

For example, a donor asked some 12 children about their ages and classes.

From the data, their classes ranged from KG1 to B3.

This data helped the donor to buy pencils and crayons for the children instead of buying pens. Learners in KG1 to lower Primary (that is Basic 3) write with pencils but not pens. Without the data about the classes of the children, the donor would have bought pens instead. The children did not need pens.

There are two kinds of data. They are primary and secondary data.

Primary data is collected from the original source for a specific purpose. The donor's data described above is a primary data. Primary data is also called first-hand data.

For example, the headteacher of Bodoma M/A J.H.S collects data on the number and gender of learners in every class of the school. The headteacher uses the data to help place some new learners he has admitted. Data collected by the headteacher is an example of primary data.

However, if the headteacher's data is further used by someone else, it becomes a secondary data to that person. For example, if the Municipal Education Director uses the headteacher's data in order to supply the right quantity of desk and school uniform, the data is a secondary data.

In this unit, our focus is on methods used to collect primary data.

26.1 Methods of Data Collection

There are various methods of collecting data. The type of data needed would determine the appropriate method to be used.

For example, to know the type of food people eat at a restaurant, you could use observation method instead of a questionnaire.

Let us discuss the various methods.

1. Questionnaires – In this method a series of questions are asked for respondents to answer. The responses are then organized to provide meaningful information. It helps to get information from many people. It can be handed out and collected later. Respondents could even post or send by e-mails.

2. Interview – this involves asking the respondents questions for them to answer orally. It could be done face to face or on telephone. Other platforms such as Zoom, Facebook and Whatsapp could also be used.

Using interviews allows the interviewer to ask further open-ended questions. Thus, based on a particular answer given, further questions could be asked in order to give a clearer understanding.

3. Observation – In observation, the observer does not ask questions for others to answer.

However, the observer spends time to look at things carefully and record what is found. Sometimes, the observe may need to count. He may even need to do a video recording or take pictures to support.

4. Experiment – this method involves the researcher change some variables and observe the effect of the changes made.

The variables manipulated are called independent-variables.

The variable that changes as the result of the manipulation are called dependent variables.

For example, if a researcher wants to find the number of guest who visit a restaurant over a period of time, he could first use 1 hour to find the number of guests. He then add another hour to make it 2 hours.

Time is being manipulated and it is the independent variable while the effect on the number of guest is the dependent variable.

5. Survey – In a survey, a sample of respondent is selected from a large population. The process of choosing individuals from the layer population to take part in the survey is called sampling.

The researcher constructs questions for respondents to answer.

Ensure that your questions are unambiguous.

Surveys enable researchers to generalise for entire population.

6. Databases – It is a source of secondary data.

The data is organised and stored so that it can be accessed electronically

from a computer system. Information could be retrieved from the database as and when needed.

7. **Electronic media or internet** – is also a way of data collection. A lot of data about different fields is found on the internet. The influx of data requires the user to be careful in collecting data on the internet. Some sources of data are not verifiable and data there could be untruthful.

Exercise: 1

What data collection method would be appropriate in each question?

1. The type of drinks to buy for a school party.
2. The gender of learners in all classes.
3. The height of germinated plant over a one-month period.
4. Do people who eat at night get big stomach?
5. The height of learners in a class.
6. The type of cars on a road.

26.2 Designing and Administering a Questionnaire

Activity:

Do a survey by producing your questions to collect information.

Study the sample below.

1. What is your name? _____
2. How old are you? _____
3. What is your favourite school subject? _____
4. What is your worst subject? _____
5. What is the most important school subject? _____
6. What is your favourite hobby? _____
7. What is your favourite day of the week? _____
8. How much do you spend on bus fares every day? _____

It is important to organise data collected to make it more meaningful.

Using tables is one way of organizing data.

A teacher at Gyakiti R/C J.H.S used the question in a survey among 8 of her students. She presented their responses in a table as shown.

Name	Age	Favourite subject	Worst subject	Important subject	Favourite hobby	Favourite week day	Bus fare daily (GH¢)
Afful	13	Science	P.E	Science	Volley	Monday	5
Baah	14	OWOP	Maths	OWOP	Football	Monday	4
Hamza	13	Maths	OWOP	Maths	Hockey	Friday	5
Buah	12	Maths	History	English	Football	Wednesday	6
Darkey	14	Maths	History	Science	Hockey	Monday	8
Oduro	14	English	Maths	English	Volley	Tuesday	4
Quatey	13	Science	P.E	Science	Football	Monday	3
Yankey	12	OWOP	Maths	Maths	Football	Thursday	4

Frequency table can be extracted for each question. Let us use the question about favourite subject to make a frequency table.

Favourite subject	Tally	Frequency
Science	//	2
OWOP	//	2
Maths	///	3
English	/	1
Total		8

From the frequency table, Maths is the favourite subject of 3 learners. It is the most favourite subject. It has the highest number of learners.

Separate tables could be drawn for each of the questions using their responses in the table.

Exercise: 2

Use the table on the responses of learners of Gyakiti R/C J.H.S to draw separate frequency tables for the rest of the responses.

Exercise: 3

Design questions for a survey on any issue in your community.

CHAPTER 27

ORGANISING AND PRESENTING DATA

STRAND 4: DATA

SUB-STRAND 1: Data and Probability

CONTENT STANDARD: B7.4.1.1 Select, justify, and use appropriate methods to collect data (quantitative and qualitative), display and analyze the data (grouped/ ungrouped) presented in frequency tables, line graphs, pie graphs, bar graphs or pictographs and use these to solve and/or pose problems.

INDICATOR B7.4.1.1.3- Organise and present data from a survey into a table and/or chart, and analyse it to solve and/or pose problems.

27.1 Using Tallies to Organise Data into Frequency Table

Let us look at how to tally data to find the frequency of given data. Frequency is the number of times a data value occurs.

Example: The marks obtained by B7 learners in a test at State Boys M/A JHS conducted by their Maths teacher Mr. M.C. Organise the data in a frequency table.

6, 4, 3, 7, 6, 6, 7, 5, 7, 2, 7, 7, 5, 6, 5, 4, 3, 8, 9, 10

Solution:

There are 20 data points. Thus there are 20 marks. In other words, 20 learners took part in the test. But the marks (data points) range from 2 to 10. Thus, there are 9 data values.

Draw a table of 9 rows and 3 columns as shown. Write all the numbers in the range in the mark column.

Marks	Tally	Frequency
2	/	1
3	//	2
4	//	2
5	///	3
6	////	4
7	~~////~~	5
8	/	1
9	/	1
10	/	1

~~6~~, ~~4~~, ~~3~~, ~~7~~, ~~6~~, ~~6~~, ~~7~~, ~~5~~, ~~7~~, ~~2~~, ~~7~~, ~~7~~, ~~5~~, ~~6~~, ~~5~~, ~~4~~, ~~3~~, ~~8~~, ~~9~~, ~~10~~

Find the first data point in the set. In our data it is 6. Draw one tally mark for 6.

How do we know we have tallied the first data point, 6? Cross it out. Look for all the 6 in the data set and use tally marks to represent them. Write the corresponding number under Frequency column.

Move to the next data point and do same.

Example: Draw a bar graph for the data below.

It is the marks obtained in a Mathematics test by students in a class.

10 7 4 5 6 8 7 6 7 5 3 4 6 5 4 5 4 6 5 6 7 6 3 4 5 8

6 7 5 9 4 6 6 1 7 7 9 5 1 5 2 7 10 8 6 7 4 1 6 6

Solution:

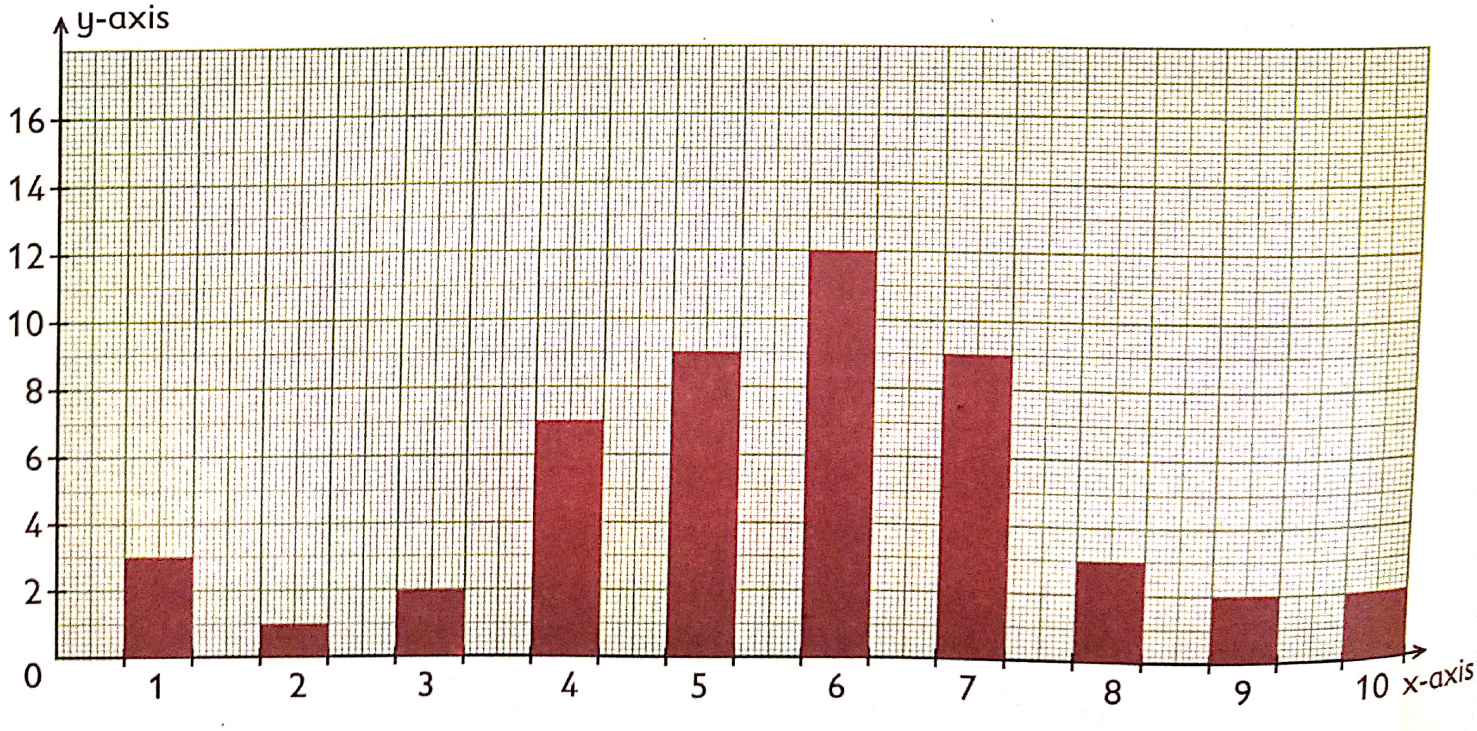
Organise data into Frequency table.

10 7 4 5 6 8 7 6 7 5 3 4 6 5 4 5 4 6 5 6 7 6 3 4 5 8

6 7 5 9 4 6 6 1 7 7 9 5 1 5 2 7 10 8 6 7 4 1 6 6

Marks	Tally	Frequency
1	///	3
2	/	1
3	//	2
4	𝍸 //	7
5	𝍸 ////	9
6	𝍸 𝍸 //	12
7	𝍸 ////	9
8	///	3
9	//	2
10	//	2

TITLE: MARKS OBTAINED BY SOME STUDENTS



Exercise: 2

Use tallies to organize the data below into frequency table. Use your table to draw a bar graph.

A survey on the number of letters in the surname of some musicians in Ghana.

3, 5, 5, 7, 4, 4, 7, 4, 5, 7, 6, 8, 3, 6, 6, 6, 6.

Use your graph to answer these questions

1. How many musicians has 6 letters in their surname?
2. How many musicians were surveyed?

Exercise: 3

The table below shows the distribution of marks of students in a class.

Mark	1	2	3	4	5	6
Frequency	5	6	5	3	4	2

1. How many students are in the class?
2. Which mark was obtained by the highest number of students?
3. What percentage of learners scored 3?

27.3 Drawing Pie Chart to Illustrate Data in a Frequency Table

A pie chart uses sectors of a circle to represent data.

To use data in a frequency table to draw a pie chart.

1. Find the total frequency.
2. Calculate the size of angle (sector) for each category.

The angle for a category = $\frac{\text{Value of category/frequency of category}}{\text{Total frequency}} \times 360^\circ$

Subject	Tally	Frequency
Physics	𝍸 𝍸 𝍸 ///	18
Biology	𝍸 𝍸 //	12
Chemistry	𝍸 𝍸 𝍸 ///	18
Elective Maths	𝍸 𝍸 𝍸 𝍸 ////	24

Solution:

Total number of students (total frequency)

= 18 + 12 + 18 + 24

= 72

A circle measures 360°

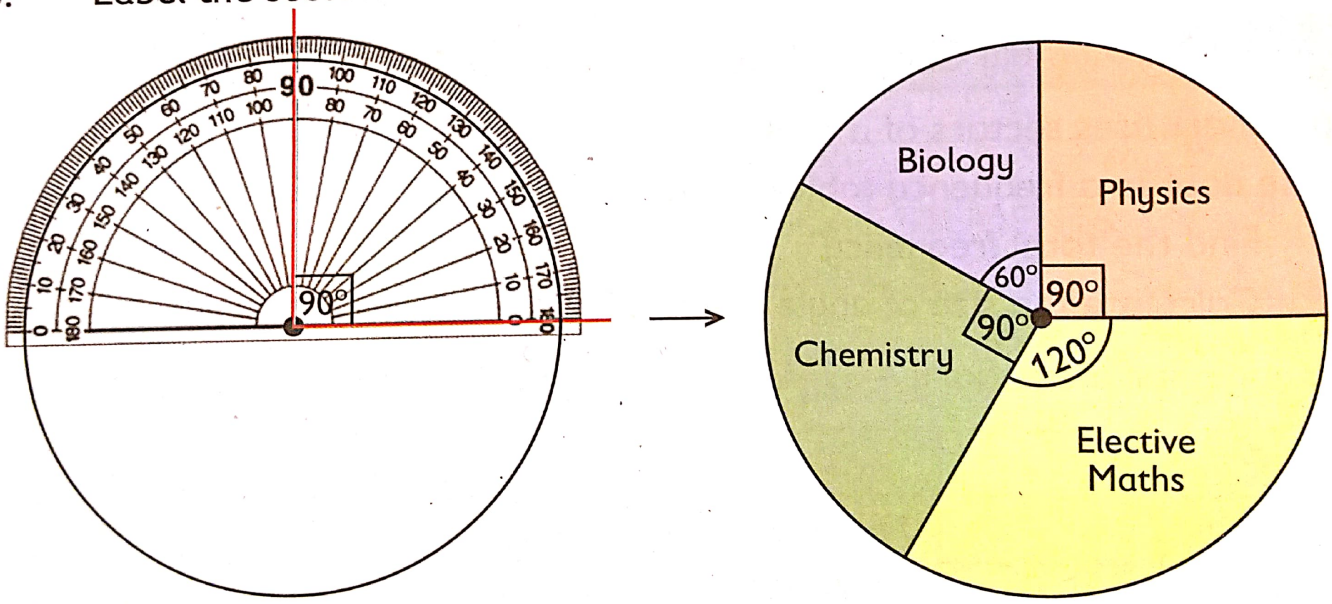
1. Calculate the sector for each subject.
2. We draw the frequency table and add one more column for angle of sector.

Subject	Tally	Frequency	Angle of sector
Physics	𝍸 𝍸 𝍸 ///	18	$\frac{18}{72} \times 360° = 90°$
Biology	𝍸 𝍸 //	12	$\frac{12}{72} \times 360° = 60°$
Chemistry	𝍸 𝍸 𝍸 ///	18	$\frac{18}{72} \times 360° = 90°$
Elective Maths	𝍸 𝍸 𝍸 𝍸 ////	24	$\frac{24}{72} \times 360° = 120°$

3. Use a compass and draw a full circle.
4. Draw a radius. With the radius as a baseline, measure the angle for the first category. In the data above, the first category or subject is physics which is 90o. Use a protractor to measure 90°.

Continue to measure the angles for the rest of the subjects.

5. Label the sectors and write the size of the angles of the sectors.



1. What is the Favourite subject among the students? From the pie chart, the subject with the largest sector is Elective Maths. Therefore, Elective Maths is the favourite subjects of most students.
2. What percentage of students like Chemistry?

Solution:

Angle for Chemistry = 90°

Total angle = 360°

Percentage of students who like Chemistry = $\frac{90°}{360°} \times 100\% = 25\%$

Example: Use tallies to organise into a frequency table the data below which was obtained by a group of learners for the number of people living in households around their houses.

3 4 2 4 3 2 2 5 4 3 2 6 3 5 4 1 2 6 3 5

5 2 4 1 5 4 2 4 3 4 2 4 4 6 2 4 3 4 2 4

Draw a pie chart to show the data.

What family size is found in most of the households?

Solution:

Organise the data into frequency table using tallies.

3 4 2 4 3 2 2 5 4 3 2 6 3 5 4 1 2 6 3 5

5 2 4 1 5 4 2 4 3 4 2 4 4 6 2 4 3 4 2 4

No/Household	Tally	Frequency	Angle of sector
1	//	2	
2	𝍸 𝍸	10	
3	𝍸 //	7	
4	𝍸 𝍸 ///	13	
5	𝍸	5	
6	///	3	

Total frequency = 2 + 10 + 7 + 13 + 5 + 3 = 40

Angle for 1 = $\frac{2}{40} \times 360° = 18°$

Angle for 2 = $\frac{10}{40} \times 360° = 90°$

Angle for 3 = $\frac{7}{40} \times 360° = 63°$

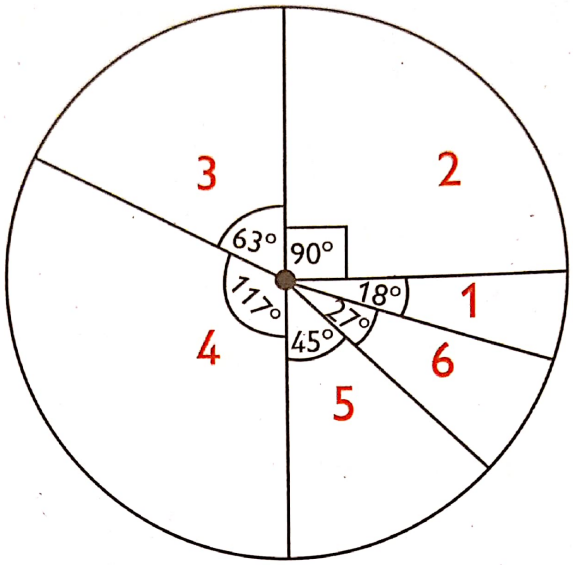
Angle for 4 = $\frac{13}{40} \times 360° = 117°$

Angle for 5 = $\frac{5}{40} \times 360° = 45°$

Angle for 6 = $\frac{3}{40} \times 360° = 27°$

Draw a circle and measure the sectors using a protractor.

Label the sectors and write the angles.



The family with the largest sector is 4 = 117°.
Families with 4 people living in household is the most.

Exercise: 4

A traffic survey gave the results shown in the table below.

Vehicle	Car	Motor bike	Bus	Bicycle
Frequency	15	12	8	25

1. Represent the information on a pie chart.
2. What percentage of the vehicles were cars?

Exercise: 5

The data below is the ages of some students.

14 17 15 17 18 17 19 14 15 19 14 17 17 18 19
14 17 15 14 17 17 19 16 14 17 15 17 18 19

1. What is the age of the oldest student?
2. What percentage of students are 18 years old?

27.4 Double Bar Graph

The double bar graph is drawn to show two sets of data. It makes it easier to compare the data.

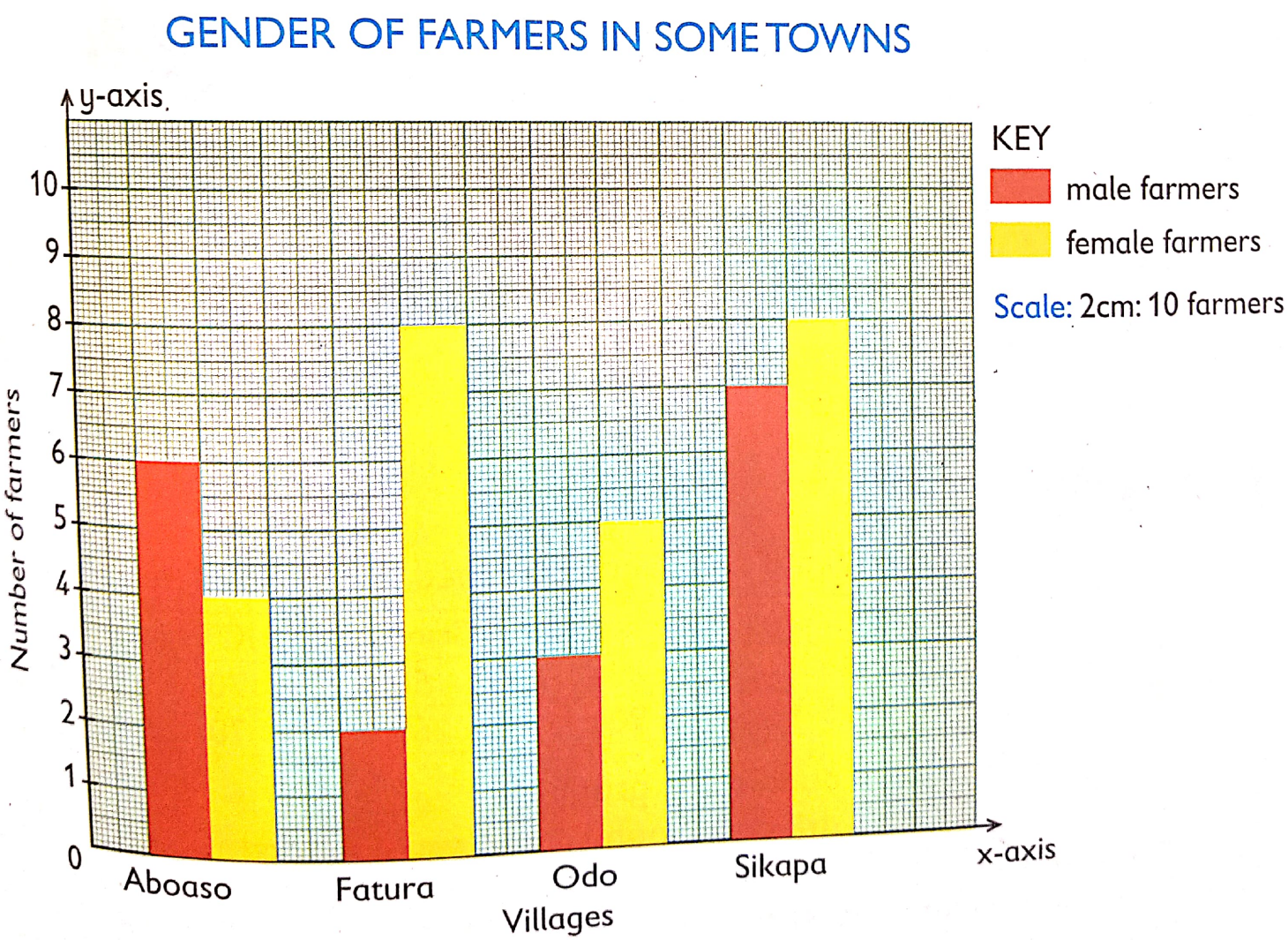
Example: The table below shows the gender of farmers in four villages.

Villages	Aboaso	Fatura	Odo	Sikapa
Male	60	20	30	70
Female	40	80	50	80

Solution: Shade the bars in a category differently. Thus, choose different colours for each category.

In this graph, let us use red to represent males and yellow to represent females.

Use a scale of 2cm to represent 10 farmers.



CHAPTER 28

MEASURES OF CENTRAL TENDENCIES

STRAND 4: DATA

SUB-STRAND 1: Data and Probability

CONTENT STANDARD: B7.4.1.2 Determine the measures of central tendency (mean, median, mode) for a given ungrouped data and use it to solve problems.

INDICATOR B7.4.1.2.1 Calculate the mean for a given ungrouped data and use it to solve problems.

B7.4.1.2.2 Calculate the median for a given ungrouped data and use it to solve problems.

A measure of central tendency is a number used to represent the center or middle of a set of data values. The mean, median and mode are three main measures of central tendency. They give information about where the bulk of scores in a particular data set are located.

28.1 Finding the Mean for a Data Set

The mean is the average of a data set.

Read the information about an End-of-term Get Together organised by Madam Rose and a group of her B.7 learners at Adadientem J.H.S.

She asked six (6) learners in a group to bring oranges so that the learners enjoy together.

The table below shows the number of oranges voluntarily brought by each of the 6 learners.

Learners	Number of oranges brought
Ramsey	4
Grace	3
Hannah	1
Julie	2
Asamoah	5
Tabuah	3

She wanted to share the oranges evenly among the 6 learners.

She put all the oranges together and shared them equally among the 6 learners.

$4 + 3 + 1 + 2 + 5 + 3 = 18$ oranges.

$18 \div 6 = 3$

Every learner got 3 oranges. This is what mathematicians call finding the mean or average of a data set.

Mean is denoted by $\bar{x}$

$$\bar{x} = \frac{x_1 + x_2 + x^3 + \cdots x_n}{x_1 + x_2 + x^3 + \cdots x_n} = \frac{\sum x}{n}$$

where $\sum x$ is the sum of all the data values and x is the individual data values or numbers.

Example 1: Calculate the mean of the following data set.

8, 9, 7, 6, 8, 10.

Solution: There are 6 numbers.

Add the numbers and divide by 6.

The mean, $\bar{x} = \frac{8+9+7+6+8+10}{6} = \frac{48}{6} = 8$

b. 12, 13, 10, 7, 6, 1, 7

Solution: There are 7 numbers. Add them and divide by 7.

Mean, $\bar{x} = \frac{12+13+10+7+6+1+7}{7} = \frac{56}{7} = 8$

c. 6, 3, 1, 4

Solution: There are 4 numbers. Add them and divide the sum by 4.

Mean, $\bar{x} = \frac{6+3+1+4}{4} = \frac{14}{4} = 3.5$

Exercise: 1

Calculate the mean of the following data sets.

1. 4, 6, 3, 7
2. 10, 15, 8, 12, 5
3. 6, 8, 3, 9, 5, 4, 7
4. 13, 10, 15, 20, 7, 13,
5. 25, 30, 20, 10, 15
6. 24, 34, 32, 16, 45, 38, 28
7. 100, 150, 200, 250
8. 9, 13, 7, 9, 7
9. 16, 15, 10, 9, 14, 8
10. 13, 12, 11, 19, 10

Finding the mean for a data set in a Frequency Table

We are going to calculate the mean using frequencies.

The frequency tells the numbers of times each data value occurs in the data set.

Mean, $\bar{x} = \frac{\sum fx}{\sum f}$

where $\sum fx$ is the sum of the products of f and x (that is fx).

$\sum f$ is the sum of the frequencies.

Example 1: Find the mean for the marks obtained out of a total of 5 in a class of 20 students.

Score	1	2	3	4	5
Frequency	3	5	4	5	3

At the beginning of the lesson, we learnt that to find the mean of a set, we add all the values and divide by the number of values.

However, in this instance if we add the numbers one by one, there would be a long addition like this

1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 4 + 4 + 5 + 5 + 5

To make it simple, add all the 1s, all the 2s, all the 3s, all the 4s and all the 5s.

We do this by multiplying each score by its frequency.

$(3 \times 1) + (5 \times 2) + (4 \times 3) + (5 \times 4) + (3 \times 5)$.

$3 + 10 + 12 + 20 + 15 = 60$

Why do we multiply 1 by 3? Because there are three 1s (the frequency of 1 is 3).

The sum of the products of the frequency and scores is 60.

The sum of the frequency, thus number of students is $3 + 5 + 4 + 5 + 3 = 20$.

The mean $= 60 \div 20 = 3$

This is simplified by adding one column in the table to find the mean.

Score (x)	Frequency (f)	fx
1	3	$(1 \times 3) = 3$
2	5	$(2 \times 5) = 10$
3	4	$(3 \times 4) = 12$
4	5	$(4 \times 5) = 20$
5	3	$(5 \times 3) = 15$

$\sum f = 20$ $\sum fx = 60$

Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{60}{20} = 3$

The mean or average score is 3.

Example 2: Calculate the mean of the ages of children at a party presented in frequency table.

Age (x)	1	3	5	6	7	8	9	10
Frequency (f)	2	5	6	10	8	5	3	1

Solution:

Age (x)	Frequency (f)	fx
1	2	2
3	5	15
5	6	30
6	10	60
7	8	56
8	5	40
9	3	27
10	1	10

$\sum f = 40$ $\sum fx = 260$

Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{260}{40} = 6.5$

The mean age is 6.5

Exercise: 2

Calculate the mean.

1. The number of goals scored by a football team in a league season.

Number of goals scored in a match	0	1	2	3	4	5
Frequency	1	7	6	4	1	1

2. The marks obtained by learners in a test.

Marks	1	2	3	4	5	6	7	8
Frequency	1	2	2	4	2	2	5	2

3. Height in metres of plants on a farm.

Marks	1	2	3	4
Frequency	1	3	2	5

4. The number of tracks on each CD.

Number of tracks	11	12	13	14	15
Frequency	1	3	0	2	4

5. The marks of some students in a test.

Marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	1	3	3	4	1	3	1	0	2

Problems Involving Calculating the Mean

Example: A shop keeper sold the following loaves of bread over the last 6 days. 25, 48, 25, 33, 59, 50. What was the average number of loaves sold each day?

Solution: Add the number of loaves for all the days.

$25 + 48 + 25 + 33 + 59 + 50 = 240$

Divide the sum by the number of days,

6.

$240 \div 6 = 40$

The average number of loaves of bread sold each day is 40.

Example: Adu scored the following marks in 4 Common Core subjects. Find the average mark. 10, 12, 14 and 16.

Solution: Add the marks and divide by 4.

$$\frac{10+12+14+16}{4} = \frac{52}{4} = 13$$

Adu's average mark is 13.

Example: Sena had the following scores in five of the Common Core subjects this term: 75, 87, 90, 88, 79. If she wishes to have an average score of 85, what must she score on the sixth text?

Solution: The score for the sixth text is unknown.

$$\frac{75+87+90+88+79+\square}{6} = 85$$

$$\frac{419+\square}{6} = 85$$

Cross multiply

$419 + \square = 85 \times 6$

$419 + \square = 510$

Group like terms

$\square = 510 - 419$

$\square = 91$

For Sena to have an average score of 85, she must score 91 in the sixth test.

Example: In a factory of 15 workers, 10 earn GH¢ 5.00 per day and each of the rest earn GH¢2.00 a day.

What is the average earning of the workers per day?

Solution: Earning of 10 workers who earn GH¢5.00 per day.

$10 \times$ GH¢5.00 = GH¢50.00

Earning of removing 5 workers of earn GH¢2.00 per day.

$5 \times$ GH¢2.00 = GH¢10.00

Total earning of all workers

= GH¢50.00 + GH¢10.00 = GH¢60.00

There are 15 workers.

Average earning of workers

= GH¢60.00 ÷ 15 = GH¢4.00

The average earning of the workers per day is GH¢4.00

Alternatively, average earning of the workers

$$= \frac{(10 \times 5)+(5 \times 2)}{15} = \frac{50+10}{15} = \frac{60}{15} = 4$$

The average earning of the workers per day is GH¢4.00

Exercise: 3

Solve the following word problems.

1. The temperature of 6 patients in a hospital are recorded as follows: 37, 34, 36, 40, 35, 34. Find the average temperature of the patients.
2. The average age of a J.H.S 1A class of 25 learners is 12 years and the average age of JHS 1B class of 25 learners is 10 years. What is the average of learners of the J.H.S 1A and JHS 1B classes?
3. The average of seven cars speeding at 75km/h, 70km/h, 63km/h, 68km/h, 72km/h, 62km/h and xkm/h is 70km/h. What is the value of x?
4. The average mass of 4 boys is 45kg. When a fifth boy joins them, the average mass changes to 40kg. What is the mass of the fifth boy?
5. The average rainfall for 6 consecutive months is 42mm. If the first five months recorded 40mm, 45mm, 36mm and 47mm. What is the rainfall

recorded in the sixth month?

6. Ali's average mark in 8 subjects is 54. If the 9th subject is added, the average mark becomes 55. Find the score of the 9th subject.

7. The mean of the numbers 15, 26, 18, 21 and y is 21. Find the value of y.

8. The mean age of 11 players is 15. The ages of nine of the players are 17, 14, 15, 18, 12, 16, 13, 17, 11. The remaining 2 players are twins born on the same day. Find the age of the twins.

9. In a class Test, Fletcher scored 72% in Biology, 68% in Chemistry and x% in Physics. If the mean mark of the three subjects is 65%, find marks scored in Physics.

10. What is the mean capacity of 4 containers, 215l, 200l, 212l, 213l.

28.2 Calculating Median

The median is also a measure of central tendency.

When a set of numbers is arranged in order, the number in the middle or center is called the median. Where there are two values in the center or middle, the median is the average or mean of these two middle values.

These are the steps to find the median of a set of 'n' numbers.

$x_1, x_2, x_3, \ldots, x_n$,

Arrange the numbers in order of magnitude. To arrange in ascending order means from smallest to biggest. To arrange in descending order means from biggest to smallest.

If n is odd, the median is the middle number.

Thus, $\frac{1}{2}(n + 1)$ th value or number.

Note that an odd number is a whole number that cannot be divided exactly into two. There is a remainder of 1 when divided by 2.

1, 3, 5, 7, 9, 11, 13 ... are odd numbers.

Example: Find the median of each set of numbers.

a. 2, 4, 3, 3, 5, 3, 6

Solution: Rearrange the numbers in ascending order of magnitude.

2, 3, 3, ③, 4, 5, 6

The numbers are 7. Thus n = 7. The median is the middle number.

$\frac{1}{2}(7 + 1)$ th

$\frac{1}{2}(8)$ th = 4th number

The 4th number is 3. Therefore, the median is 3.

b. 19, 29, 36, 15 and 20

Solution: Rearrange the numbers in ascending order.

15, 19, 20, 29, 36.

There are 5 values or numbers. Thus $n = 5$ and 5 is an odd number.

The median = $\frac{1}{2}(5 + 1)$ th value.

= $\frac{1}{2} \times 6$th = 3rd value.

The third value in the set is 20. Therefore, 20 is the median.

c. 7, 3, 3, 7, 4, 3, 8, 7, 5

Solution:

Rearrange the values in ascending order.

3, 3, 3, 4, 5, 7, 7, 7, 8

There are 9 values. Thus n = 9 and 9 is an odd number.

The median = $\frac{1}{2}(9 + 1)$ th

$= \frac{1}{2} \times 10\text{th} = 5\text{th}$

The 5th number on the array is 5.

Therefore, the median is 5.

In finding the median, remember to arrange the numbers in order of magnitude before you find the middle number.

For example, in example c, if we had just chosen the 5th number without arranging, the median would have been 4 instead of 5.

Exercise: 4

Find the median of the following.

1. 2, 3, 5, 5, 4, 3, 2
2. 10, 9, 7, 13, 15
3. 19, 20, 18, 18, 16, 21, 17
4. 3, 2, 1, 5, 6, 4, 7, 9, 8
5. 6, 9, 4
6. 2, 4, 10, 3, 5, 9, 7
7. 5, 7, 2, 9, 11, 10, 2, 12, 2, 9, 4
8. 8, 7, 4, 3, 9, 11, 10
9. 1, 5, 6, 4, 7, 2, 8
10. 21, 20, 23, 18, 19

Let us continue to look at how to find the median when the number of values in a data set is even.

Remember that an even number is a number that can be divided exactly into 2. A number which is divisible by 2 and generates a remainder of 0.

If the number of values in a data set is even, the median is the mean or average of the two middle numbers.

Thus, the mean of $\frac{1}{2}(n)$ th and $[\frac{1}{2}(n)+1]$ th values.

First arrange the numbers in order of magnitude before finding the mean of the two middle numbers.

Example: Find the median of the following data set

a. 3, 7, 8, 6, 4, 8, 4, 6

Solution: Arrange the numbers in ascending order

3, 4, 4, 6, 6, 7, 8, 8

There are 8 numbers, that is n = 8 which is even.

The median is the mean of the two middle numbers.

$\frac{1}{2}(n)$ th = 1/2 (8) th = 4th number which is 6.

$[\frac{1}{2}(8)+1]$th = (4 + 1) th = 5th

The 5th number is also 6.

The mean of the 4th and 5th numbers is

$$\frac{6+6}{2} = \frac{12}{2} = 6$$

The median is 6.

b. 17, 15, 12, 8, 13, 14

Solution: Arrange the number in ascending order 8, 12, 13, 14, 15, 17.

There are 6 numbers. 6 is even.

$\frac{1}{2}(n)$ th $= \frac{1}{2}(6)$ th = 3rd.The third number is 13.

$[\frac{1}{2}(n)+1] = [\frac{1}{2}(6)+1]$th = (3 + 1) th = 4th.

The number is 14

The mean for the 3rd and 4th numbers is $\frac{13+14}{2} = \frac{27}{2} = 13.5$

The median is 13.5

c. 24, 20, 21, 19, 26, 30, 28, 27

Solution: Arrange the numbers in ascending order

19, 20, 21, 24, 26, 27, 28, 30.

There are 8 numbers. 8 is an even number.

Find the mean of the $\frac{1}{2}(n)$ th number and $[\frac{1}{2}(n)+1]$ th number.

$\frac{1}{2}(n)$ th = $\frac{1}{2}(8)$ th = 4th number.

The 4th number is 24.

$[\frac{1}{2}(n)+1]$ th = $[\frac{1}{2}(8)+1]$ th = 5th number. The 5th number is 26.

The mean of 24 and 26 =

$$\frac{24+26}{2} = \frac{50}{2} = 25$$

The median is 25.

Exercise: 5

Find the median of the following.

1. 24, 20, 42, 40, 32, 28
2. 17, 12, 15, 16, 8, 13, 18, 14
3. 45, 12, 75, 81, 54, 51, 24, 67, 19, 39
4. 45, 47, 28, 44, 42, 30
5. 45, 29, 17, 12, 13, 28,
6. 2, 3, 7, 8, 5, 10
7. 30, 35, 32, 38, 31, 34
8. 13, 16, 19, 12, 15, 18
9. 8, 9, 14, 14, 18, 12, 6, 17, 14, 8
10. 8, 6, 2, 2, 4, 8

Exercise: 6

Find the mean.

1. 4, 2, 6, 3, 1
2. 13, 19, 16, 10, 12, 14
3. 8, 7, 8, 6, 8, 4, 2
4. 43, 48, 40, 37, 29, 48, 40
5. 2, 5, 3, 9, 6, 4, 5, 7
6. 38, 35, 40, 37, 39, 38
7. 22, 41, 35, 63, 82, 74
8. 200, 250, 220, 750
9. 39, 45, 63, 89, 25, 32, 39
10. 100, 115, 110, 100, 120

Finding the Median for a Data Set in Frequency Table

We can also find the median for data in a frequency table.

STEPS:

1. Find the total or sum of the frequencies that is $\sum f$ or n.
2. If the total frequency ($\sum f$ or n) is an odd number, the median is $\frac{1}{2}(n + 1)$.

And if $\sum f$ or n is an even number, the median is the mean or average of the two middle values.

Thus, $\frac{1}{2}(\sum f)$th and $[\frac{1}{2}(\sum f)+1]$ th

Example: Find the median.

a. The ages of children at a party.

Age	1	3	5	6	7	8	9
Frequency	2	5	6	10	8	5	3

Solution:

a. Find the total frequency.

$\sum f$ or n = 2 + 5 + 6 + 10 + 8 + 5 + 3 = 39. We know that 39 is an odd number.

The median = $\frac{1}{2}(\sum f + 1)$ th = $\frac{1}{2}(39 + 1)$ = $\frac{1}{2}(40)$ = 20th value or number.

Starting from 2, add the frequencies until you get the 20th value or number. The 20th value (age) is 6.

b. The marks obtained in a Mathematics test.

Score	1	2	3	4	5
Frequency	2	6	4	5	3

Solution

Find the total frequency ($\sum f$ or n)

$\sum f$ = 2 + 6 + 4 + 5 + 3 = 20

20 is an even number

The median is the mean of the two middle values.

$\frac{1}{2}(\sum f)$ th and $[\frac{1}{2}(\sum f)+1]$ th

$\frac{1}{2}(20)$ th = 10th value

$\frac{1}{2}(20) + 1 = 10 + 1 = 11$th value.

Add the frequencies, starting from 2 until you reach 10th and 11th values. The 10th and 11th values are both 3. The median score is 3.

The height of some B7 learners at PSK M/A Basic School.

Height (m)	150	154	155	159	165	169	174
Frequency	1	1	3	1	2	1	1

Solution

Find the total frequency ($\sum f$ or n).

$\sum f = 1 + 1 + 3 + 1 + 2 + 1 + 1 = 10$

10 is an even number.

The median height is the mean of the two middle values.

$\frac{1}{2}(10)$ th = 5th $\frac{1}{2}[(10)+1]$ th = 6th

It is mean of the 5th and 6th values.

Add the frequencies, starting from 1 until you reach the ages that corresponds to the 5th and 6th values or numbers.

The age for the 5th number is 155.

The age for the 6th number is 159.

Therefore, the median height is

$$\frac{155+159}{2} = 157$$

The median height is 157cm.

Exercise: 7

Find the median for the following frequency tables.

1. Ages of kids in park.

Age	6	7	8	9	10
Frequency	3	2	4	1	1

2. Marks scored by some students in a test.

Marks	3	4	5	6	7	8
Frequency	6	9	5	4	3	3

3. Marks scored by some learners in a Science test.

Marks	1	2	3	4	5	6	7	8	9	10
Freq.	2	2	3	3	4	2	2	0	1	1

4. The mass of some items in a shop.

Mass (kg)	5	7	10	11	12
Frequency	3	1	5	2	5

5. Marks obtained by 20 learners in a test.

Marks	1	2	3	4	5	6	7
Frequency	1	3	5	6	2	1	2

Exercise: 8

Use the frequency tables to find the median.

1. Temperature of some patients.

Temperature (°C)	30	31	32	33	34	35
Frequency	1	4	2	1	1	1

2. Ages of kids in park.

Age (yrs.)	12	13	14	15	16	17	18	19
Freq.	3	4	1	2	5	3	2	2

3. The marks obtained by 8 learners.

Marks	1	2	3	4	5
Frequency	1	3	2	1	1

4. The scores of some students in an examination.

Marks	2	3	4	5	6
Frequency	2	4	5	3	1

5. The sizes of shoes made by a factory.

Size	37	38	39	40	41	42	43
Frequency	4	6	9	15	14	7	5

28.3 The Mode

The mode of a data set is the value that appears most often or most frequently. In a frequency table, the mode is the value with the highest frequency.

Example: Find the mode of the following numbers.

1. 3, 5, 1, 1, 2, 3, 2, 4, 3

Solution: The mode is 3. It occurs most frequent.

2. 32, 28, 30, 32, 31, 29, 32

Solution: 32 is the mode. It appears most often.

3. 10, 11, 9, 8, 10, 8, 9, 10, 10

Solution: 10 is the mode. It appears most often.

Finding the mode from a frequency table

In a frequency distribution table, the mode is the item which has the highest frequency.

Example 4: Find the modal age of some learners.

Age	10	11	12	13	14
Frequency	3	5	8	2	4

Solution: Check from the frequency row. Identify the highest number. The mode is the item of the highest frequency. The modal age is 12. It has the highest frequency.

Example 5: Find the modal mark.

Mark	1	2	3	4	5
Frequency	0	3	2	8	7

Solution: The modal mark is 4. It has the highest frequency.

Exercise: 9

Find the mode in the following.

1. 10, 9, 6, 6, 4, 3, 7, 6
2. 1, 0, 1, 2, 0, 5, 3, 1, 2, 1
3. 4, 9, 8, 6, 7, 8, 5, 8, 6, 8
4. 1, 1, 3, 3, 4, 1, 2, 1, 5, 1
5. 120, 114, 193, 120, 142, 120, 118
6. 3, 9, 3, 2, 1, 5, 3, 4, 3
7. 2, 2, 3, 1, 0, 1, 2, 5, 2
8. 45, 40, 42, 40, 45, 45, 44, 45
9. 18, 12, 13, 17, 12, 13, 12, 14, 12
10. 36, 35, 37, 36, 37, 38, 37

Exercise: 10

Find the mode in the following frequency tables.

Number of goals	0	1	2	3	4
Frequency	10	7	8	15	4

Age	8	9	10	11
Frequency	5	10	6	9

Temperature (°C)	23	24	25	26
Frequency	7	31	42	30

Marks	0	1	2	3	4	5
Frequency	4	3	9	2	8	4

Height	170	175	180	185	190
Frequency	7	12	3	2	1

CHAPTER 29 PROBABILITY

STRAND 4: HANDLING DATA

SUB-STRAND 2: Chance or Probability

CONTENT STANDARD: B7.4.2.1 Identify the sample space for a probability experiment involving single events and express the probabilities of given events as fractions, decimals, percentages and/or ratios to solve problems.

INDICATOR B7.4.2.1.1 Demonstrate understanding of likelihood of a single outcome occurring by providing examples of events that are impossible, possible, or certain from personal contexts.

B7.4.2.1.2 Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain.

B7.4.2.1.3 Calculate the probability of the event and express the probability as fractions, decimals, percentages and/or ratios.

29.1 Likelihood of a Single Outcome Occuring

In our everyday lives, we try to think or understand whether or not something might happen. We make decisions based on the chance or probability that an event may or may not happen. Probability is a way mathematicians use to measure the likelihood of events happening.

The terms, outcome, event, experiment need to be explained.

Outcomes are the different results that are obtained from an experiment. For example, when a coin is tossed once, the results are head or tail. These results are the outcomes of undertaking experiment of throwing a coin once.

An event is any set of outcomes. For example when a die is rolled, the event "rolling an even number" has the set of outcomes 2, 4 and 6.

Exercise: 1

Botwe rolled a ludo die once. What outcomes make up the event of Botwe obtaining?

1. a multiple of 2?
2. a multiple of 3?
3. a factor of 4?
4. a factor of 6?
5. a prime number?
6. an even number?
7. a multiple of 4?
8. a number greater than 6?

The likelihood of any event happening is somewhere between impossible and certain.

It could be shown on a probability line. The probability line is a line segment showing how likely or unlikely an even would happen. See Figure 29.1

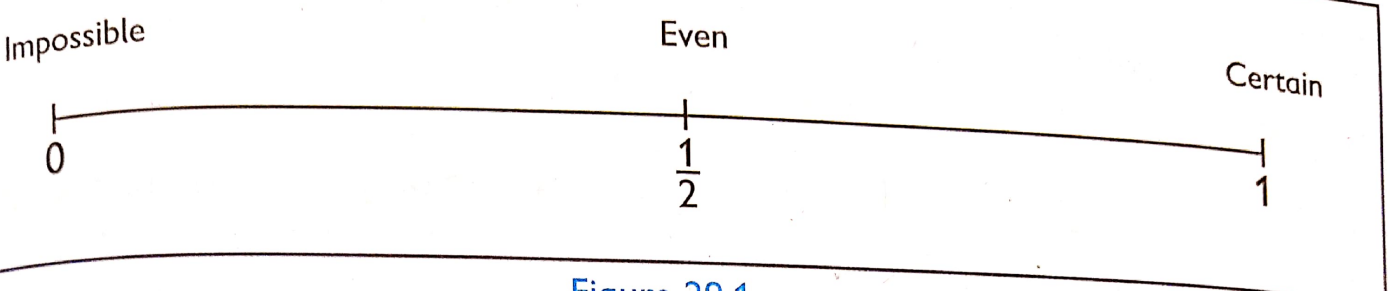


Figure 29.1

Let us describe and show each outcome using words like: impossible, possible or certain.

A. The dog will fly tomorrow. This is impossible to happen because a dog does not have wings to fly. Hence on the probability line it is shown at the extreme left.

B. A coin tossed lands Head side up (Possible).

There is even chance of this happening. A coin tossed has only two outcomes. It would be Head side up or Tail side up.

C. Ghana will still be an African country tomorrow.

This certain because Ghana cannot become a country of a different continent.

These outcomes are shown on the probability line below.

Study it and use the understanding to predict the outcome of other experiments.

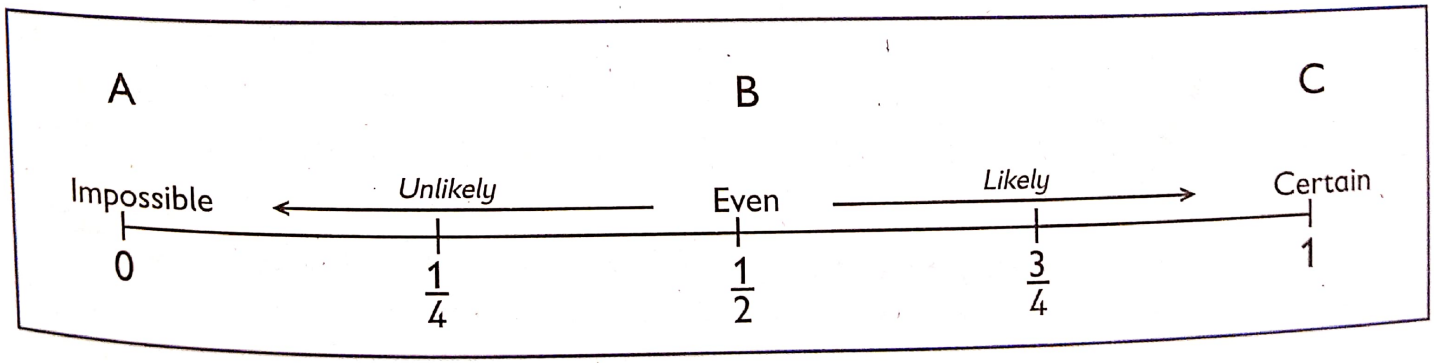


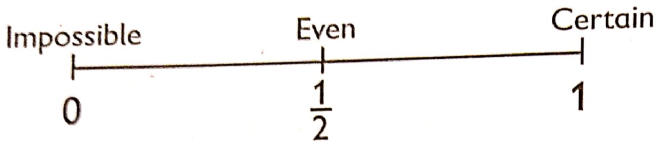
Figure 29.2

An Event or outcome which is impossible has a value of zero (0).

An Event or outcome which is certain to happen is one (1). The probability of choosing a red ball although possible but unlikely. The probability of selecting a blue ball is likely. This is because there are more blue balls and only 1 red ball.

Exercise: 2

Show the probability of the following outcomes by placing the letters of the outcomes on a probability line.



A. A tossed coin landing Tail side up.
B. The sun will rise in the East next tomorrow.
C. It will rain tomorrow.
D. A new born baby will be a girl.
E. Rolling a 7 on a die.
F. The day after Tuesday will be Wednesday.

29.2 Classifying the Likelihood of a Single Outcome

Your teacher will put you in groups to discuss the following outcomes of throwing a die.

Use words like impossible, possible, or certain.

A. Throwing a single die

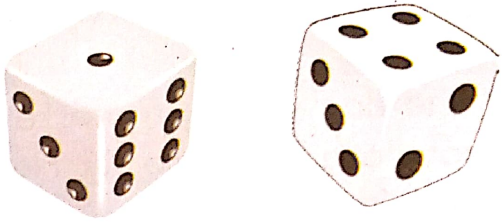
Look at ludo die.

It has six faces with dots. The dots 1, 2, 3, 4, 5 and 6 are found on the faces. When the die is thrown at any point in time, one of the number of dots shows up, that is there are six possible outcomes.

Let us look at the likelihood of the following outcomes.

1. Obtaining the number 1. Since there is 1 on one of the faces. It is possible for 1 to show up.
2. Obtaining the number 7. Since there is no 7 on the die, it is impossible for 7 to show up.
3. Obtaining the number 4. Since there is 4 on the die, it is possible for 4 to show up.

B. Throwing 2 dice



When two dice are thrown, there are six possible outcomes for each die. The result or outcome of one die does not depend on that of the other. They are independent.

1. The likelihood of obtaining a total of 12.

It is possible. This is because each could have 6 as the outcome. 6 + 6 = 12.

2. Obtaining a total of 13. This is impossible because even if one die shows 6, you need 7 to get a sum of 13 and 7 is not on the face of a die to show up

Thus, any sum more than 12 is impossible to happen.

29.3 Calculating and Expressing Probability of Events As Fractions, Decimals, Percentages and or Ratios

We have already learnt that if the probability of an event is 0, it is impossible to happen.

If it is 1, it is certain to happen. If it is $\frac{1}{2}$, it is (that is may or may not happen).

Probability may be represented in the form of fraction, decimal, percent and ratio.

The probability in the probability line Figure 29.2 could be expressed in decimal, percent and ratio forms as well.

Fraction	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Decimal	0	0.25	0.50	0.75	1.00
Percent	0	25%	50%	75%	100%
Ratio	0	1:4	1:2	3:4	1:1

In an experiment, Emefa threw a single die.

Let us use the worksheet below to calculate the probabilities of various events happening. Express the probabilities in other forms such as fractions, decimal, percent and ratio.

The probability of rolling:	Fractions	Decimals	Percentages	Ratios
1. factors of 60				
2. a multiple of 3				
3. factors of 2				
4. a 3 or greater				
5. factors of 8				

When a single die is thrown once, there are six possible outcomes: 1, 2, 3, 4, 5, 6. The die could show any of the six numbers.

The probability of an event, E occurring is given by $P(E) = \frac{n(E)}{n(U)}$ where $n(E)$ is the number of elements in E and $n(u)$ is the number of possible outcome.

Solution:

Possible outcomes = {1, 2, 3, 4, 5, 6} $n(u) = 6$.

Probability of rolling factors of 60.

The factors of 60 among the possible outcomes are 1, 2, 3, 4, 5 and 6.

Therefore, $n(E) = 6$

$$P(E) = \frac{n(E)}{n(U)} = \frac{6}{6} = \frac{1}{1} = 1$$

Convert $\frac{1}{1}$ into decimals

$\frac{1}{1} = 1.0$

Change 1/1 into percentage

$\frac{1}{1} \times 100\% = 100\%$

Express $\frac{1}{1}$ in ratio

$\frac{1}{1} = 1:1$

2. Probability of rolling a multiple of 3.

The multiples of 3 among the possible outcomes are 3 and 6.

Therefore, n (E) = 2.

$$P(E) = \frac{n(E)}{n(U)} = \frac{2}{6} = \frac{1}{3}$$

Change 1/3 into decimals by dividing the numerator by the denominator.

$\frac{1}{3} = 1 \div 3$

$$\begin{array}{r} 0.33 \\ 3\overline{)\,10} \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$

$= 0.33$

Change $\frac{1}{3}$ into percentage by multiplying the fraction by 100.

$\frac{1}{3} \times 100 = \frac{1 \times 100}{3} = \frac{100}{3} = 33.3\%$

Express $\frac{1}{3}$ in ratio.

$\frac{1}{3} = 1:3$

3. Probability of rolling factors of 2.

The factors of 2 among the possible outcomes are 1 and 2.

Therefore, n (E) = 2

$$P(E) = \frac{n(E)}{n(U)} = \frac{2}{6} = \frac{1}{3}$$

Change $\frac{1}{3}$ into decimals, percentage and ratio as done in Question 2.

4. Probability of rolling a 3 or greater.

Rolling a 3 or greater among the outcomes are 3, 4, 5, 6. Therefore, n (E) = 4

$$P(E) = \frac{n(E)}{n(U)} = \frac{4}{6} = \frac{2}{3}$$

Change $\frac{2}{3}$ into decimal by dividing 2 by 3.

$$\begin{array}{r} 0.66 \\ 3\overline{)\,20} \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

$= 0.66$

Change $\frac{2}{3}$ into percentage by multiplying by 100.

$\frac{2}{3} \times 100 = \frac{200}{3} = 66.66\%$

Express $\frac{2}{3}$ in ratio = 2:3

5. Probability of rolling factors of 8.

Factors of 8 among the outcomes are 1, 2 and 4.

Therefore, n (E) = 3

$$P(E) = \frac{n(E)}{n(U)} = \frac{3}{6} = \frac{1}{2}$$

Change $\frac{1}{2}$ into decimals.

$$\begin{array}{r} 0.5 \\ 2\overline{)\,10} \\ -10 \\ \hline 00 \end{array}$$

$= 0.5$

Change $\frac{1}{2}$ into percentages. $= \frac{1}{2} \times 100 = \frac{100}{2} = 50\%$

Express $\frac{1}{2}$ in ratio form. $= \frac{1}{2} = 1:2$

Use the answers to complete the table on Emefa's experiment of rolling a single die once.

The probability of rolling:	Fractions	Decimals	Percentages	Ratios
1. factors of 60	$\frac{1}{1} = 1$	1	100	1:1
2. a multiple of 3	$\frac{1}{3}$	0.33	33.33	1:3
3. factors of 2	$\frac{1}{3}$	0.33	33.33	1:3
4. a 3 or greater	$\frac{2}{3}$	0.66	66.66	2:3
5. factors of 8	$\frac{1}{2}$	0.50	50	1:2

Exercise: 3

Calculate the probability and complete the table on an experiment of rolling a single die once.

The probability of rolling:	Fractions	Decimals	Percentages	Ratios
1. factors of 6				
2. a 4 or smaller				
3. a 3 or smaller				
4. divisors of 30				
5. an even number				
6. divisors of 15				
7. an odd number				
8. a multiple of 6				

Exercise: 4

A bag contains 12 shapes as shown. Calculate the probabilities of the given events. Express them in fraction, ratio, percentage and decimal forms.